MODERN PHYSICS

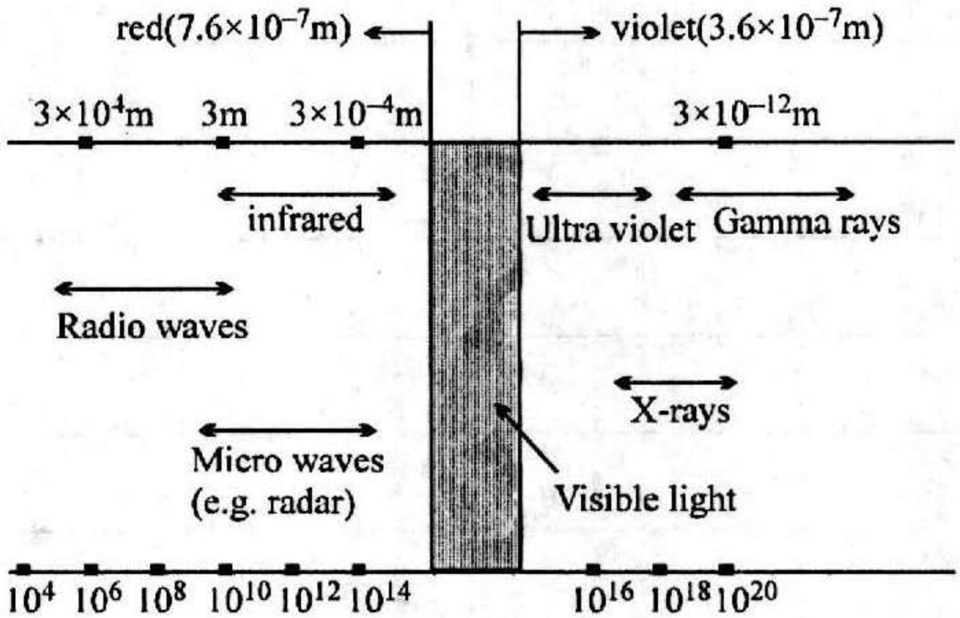
Cathode rays :

- Generated in a discharge tube in which a high vaccum is maintained.
- They are electrons accelerated by high potential difference(10 to 15 kV)
- K.E. of C.R. particle accelerated by a p.d. V is $eV = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
- Can be deflected by Electric & magnetic fields.

Electromagnetic Spectrum :

Ordered arrangement of the big family of electro magnetic waves (EMW) either in ascending order of frequencies or decending order of wave lengths.

Speed of E.M.W. in vacuum : $c = 3 \times 10^8$ m/s $= \nu\lambda$



PLANK'S QUANTUM THEORY

A beam of EMW is a stream of discrete packets of energy called **PHOTONS**; each photon having a frequency ν and energy $= E=h\nu$

where h = planck's constant = 6.63×10^{-34} J-s.

- According to Planck the energy of a photon is directly proportional to the frequency of the radiation.

$$E = \frac{hc}{\lambda} = \frac{12400}{\lambda} eV - \mathring{A} \quad \left[\because \frac{hc}{e} = 12400(\mathring{A} - eV)\right]$$

- Effective mass of photon $m = \frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$ i.e. $m \propto \frac{1}{\lambda}$

  So mass of violet light photon is greater than the mass of red light photon. ($\because \lambda_R > \lambda_V$)

- Linear momentum of photon $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

- **Intensity of light** : $I = \frac{E}{At} = \frac{P}{A}$ ...(i)

  Here P = power of source, A = Area, t = time taken

  E = energy incident in t time = $Nh\nu$ N = no. of photon incident in t time

  Intensity $I = \frac{N(h\nu)}{At} = \frac{n(h\nu)}{A}$ ...(ii) $\left[\because n = \frac{N}{t} = \text{no. of photon per sec.}\right]$

  From equation (i) and (ii), $\frac{P}{A} = \frac{n(h\nu)}{A} \Rightarrow n = \frac{P}{h\nu} = \frac{P\lambda}{hc} = 5 \times 10^{24}\, J^{-1} m^{-1} \times P \times \lambda$

- **Force exerted on perfectly reflecting surface**

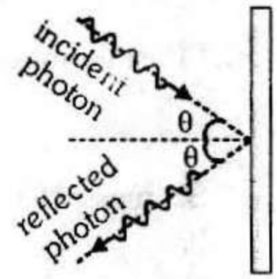
  $\therefore \quad F = n\left(\frac{2h}{\lambda}\right) = \frac{2P}{c}$ and Pressure$= \frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c} \left[\because I = \frac{P}{A}\right]$

- **Force exerted on perfectly absorbing surface**

$$F = \frac{P}{c} \left(\because n = \frac{P\lambda}{hc}\right) \text{ and } \text{Pressure} = \frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}$$

- **When a beam of light is incident at angle θ on perfectly reflector surface**

$$F = \frac{2IA\cos^2\theta}{c}$$



- **When a beam of light is incident at angle θ on perfectly absorbing surface** $F = \frac{IA\cos\theta}{c}$

PHOTO ELECTRIC EFFECT

The phenomenon of the emission of electrons, when metals are exposed to light (of a certain minimum frequency) is called photo electric effect.

Results :

- Can be explained only on the basis of the quantum theory (concept of photon)
- Electrons are emitted if the incident light has frequency $\nu \geq \nu_0$ (threshold frequency). Emission of electrons is independent of intensity . The wave length corresponding to ν_0 is called threshold wave length λ_0.
- ν_0 is different for different metals.
- Number of electrons emitted per second depends on the intensity of the incident light.

EINSTEINS PHOTO ELECTRIC EQUATION :

Photon energy = KE_{max} of electron + work function

$h\nu = KE_{max} + \phi$

ϕ = Work function = energy needed by the electron in freeing itself from the atoms of the metal $\phi = h\nu_0$

STOPPING POTENTIAL OR CUT OFF POTENTIAL :

The minimum value of the retarding potential to prevent electron emission is

$$eV_{cut\ off} = (KE)_{max}$$

Note: The number of photons incident on a surface per unit time is called photon flux.

WAVE NATURE OF MATTER :

Beams of electrons and other forms of matter exhibit wave properties including interference and diffraction with a de Broglie wave length given by $\lambda = \frac{h}{p}$ (wave length of a praticle).

- **De Broglie wavelength associated with moving particles**

If a particle of mass m moving with velocity v.

Kinetic energy of the particle $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

momentum of particle $p = mv = \sqrt{2mE}$

the wave length associated with the particles is $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

- **De Broglie wavelength associated with the charged particles :-**

- **For an Electron** $\lambda_e = \frac{12.27 \times 10^{-10}}{\sqrt{V}} m = \frac{12.27}{\sqrt{V}} \mathring{A}$ so $\lambda \propto \frac{1}{\sqrt{V}}$

- **For Proton** $\lambda_p = \frac{0.286 \times 10^{-10}}{\sqrt{V}} m = \frac{0.286}{\sqrt{V}} \mathring{A}$

- **For Deuteron** $\lambda_d = \frac{0.202}{\sqrt{V}} \mathring{A}$

- **For α Particles** $\therefore \lambda_\alpha = \frac{0.101}{\sqrt{V}} \mathring{A}$

ATOMIC MODELS :

(a) Thomson model : (Plum pudding model)

- Most of the mass and all the positive charge of an atom is uniformly distributed over the full size of atom (10^{-10} m).
- Electrons are studded in this uniform distribution .
- Failed to explain the large angle scattering α - particle scattered by thin foils of matter.

(b) Rutherford model : (Nuclear Model)

- The most of the mass and all the positive charge is concentrated within a size of 10^{-14} m inside the atom. This concentration is called the atomic nucleus.
- The electron revolves around the nucleus under electric interaction between them in circular orbits.
- An accelerating charge radiates the nucleus spiralling inward and finally fall into the nucleus, which does not happen in an atom. This could not be explained by this model.

(c) Bohr atomic model : Bohr adopted Rutherford model of the atom & added some arbitrary conditions. These conditions are known as his postulates

- The electron in a stable orbit does not radiate energy.
- A stable orbit is that in which the angular momentum of the electron about nucleus is an integral (n) multiple of $\frac{h}{2\pi}$ i.e. $mvr = n\frac{h}{2\pi}$; n=1, 2, 3, ..($n \neq 0$).
- The electron can absorb or radiate energy only if the electron jumps from a lower to a higher orbit or falls from a higher to a lower orbit.
- The energy emitted or absorbed is a light photon of frequency ν and of energy. **$E = h\nu$**

For hydrogen atom : (Z = atomic number = 1)

- L_n = angular momentum in the n^{th} orbit = $n\frac{h}{2\pi}$.
- r_n = radius of n^{th} circular orbit = (0.529 Å) $n^2 \Rightarrow r_n \propto n^2$.
- E_n = Energy of the electron in the n^{th} orbit = $\frac{-13.6\text{ eV}}{n^2} \Rightarrow E_n \propto \frac{1}{n^2}$.

  Note : Total energy of the electron in an atom is negative, indicating that it is bound.

  Binding Energy $(BE)_n = -E_n = \frac{13.6\text{ eV}}{n^2}$

- $E_{n2} - E_{n1}$ = Energy emitted when an electron jumps from n_2^{th} orbit to n_1^{th} orbit ($n_2 > n_1$).

$$\Delta E = (13.6\text{ eV})\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$\Delta E = h\nu$; ν = frequency of spectral line emitted .

$$\frac{1}{\lambda} = \text{wave no. [no. of waves in unit length (1m)]} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where R = Rydberg's constant, for hydrogen = $1.097 \times 10^7\ m^{-1}$

- For hydrogen like atom/species of atomic number Z :

$$r_{nz} = \frac{\text{Bohr radius}}{Z} n^2 = (0.529\ Å)\frac{n^2}{Z};\ E_{nz} = (-13.6)\frac{Z^2}{n^2}\ eV$$

$R_z = RZ^2$; Rydberg's constant for element of atomic no. Z .

Note : If motion of the nucleus is also considered , then m is replaced by μ. Where μ= reduced mass of electron - nucleus system = mM/(m+M)

$$\text{In this case } E_n = (-13.6\ eV)\frac{Z^2}{n^2}\cdot\frac{\mu}{m_e}$$

Spectral series :

- **Lyman Series :** (Landing orbit n = 1) .

Ultraviolet region $\bar{v} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$; $n_2 > 1$

- **Balmer Series:** (Landing orbit n = 2)

Visible region $\bar{v} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$; $n_2 > 2$

- **Paschan Series :** (Landing orbit n = 3)

In the near infrared region $\bar{v} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$; $n_2 > 3$

- **Bracket Series :** (Landing orbit n = 4)

In the mid infrared region $\bar{v} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]$; $n_2 > 4$

- **Pfund Series :** (Landing orbit n = 5)

In far infrared region $\bar{v} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]$; $n_2 > 5$

In all these series $n_2 = n_1 + 1$ is the α line

$= n_1 + 2$ is the β line

$= n_1 + 3$ is the γ line.......etc.

where n_1 = Landing orbit

- **Total emission spectral lines**

From $n_1 = n$ to $n_2 = 1$ state $= \frac{n(n-1)}{2}$

From $n_1 = n$ to $n_2 = m$ state $= \left(\frac{(n-m)(n-m+1)}{2}\right)$

Excitation potential of atom :

Excitation potential for quantum jump from $n_1 \rightarrow n_2 = \frac{E_{n2} - E_{n1}}{\text{electron charge}}$

Ionization energy of hydrogen atom :

The energy required to remove an electron from an atom. The energy required to ionize hydrogen atom is = $0 - (-13.6) = 13.6$ eV.

Ionization Potential :

Potential difference through which a free electron is moved to gain ionization energy $= \frac{-E_n}{\text{electronic charge}}$

X - RAYS :

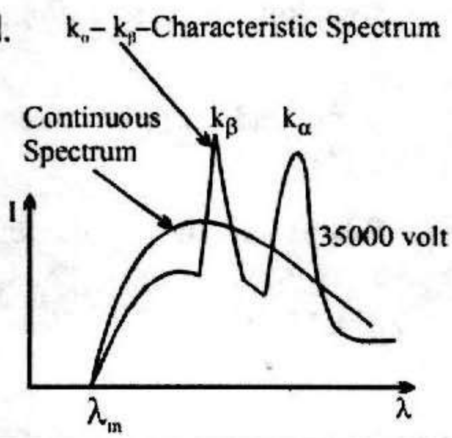
- X–rays are produced by bombarding high speed electrons on a target of high atomic weight and high melting point.
- Short wavelength (0.1 Å to 10 Å) electromagnetic radiation.
- Are produced when a metal anode is bombarded by very high energy electrons
- Are not affected by electric and magnetic field.
- They cause photoelectric emission.

Characteristics equation $eV = h\nu_m$

e = electron charge;

V= accelerating potential

ν_m = maximum frequency of X – radiation



- Intensity of X - rays depends on number of electrons hitting the target .
- Cut off wavelength or minimum wavelength, where V (in volts) is the p.d. applied to the tube $\lambda_{min} \cong \frac{12400}{V} \text{Å}$
- Continuous spectrum due to retardation of electrons .

- **Characteristic X-rays**

For K_α, $\lambda = \frac{hc}{E_K - E_L}$ For K_β, $\lambda = \frac{hc}{E_L - E_M}$

- **Moseley's law for characteristic spectrum :**

Frequency of characteristic line $\sqrt{v} = a(Z - b)$

Where a, b are constant, for K_α line b = 1

Z = atomic number of target

v = frequency of characteristic spectrum

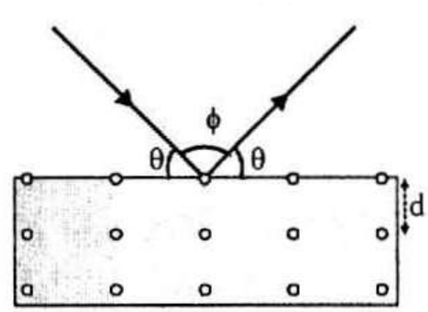
b = screening constant (for K– series b=1, L series b=7.4),

a = proportionality constant

Bohr model	**Moseley's correction**
1. For single electron species	**1.** For many electron species
2. $\Delta E = 13.6Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]eV$	**2.** $\Delta E = 13.6\ (Z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]eV$
3. $v = RcZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$	**3.** $v = Rc(Z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
4. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$	**4.** $\frac{1}{\lambda} = R(Z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Diffraction of X–ray

Diffraction of X–ray take place according to Bragg's law $2d\sin\theta = n\lambda$



d = spacing of crystal plane or lattice constant or distance between adjacent atomic plane

θ = Bragg's angle or glancing angle

ϕ = Diffracting angle n = 1, 2, 3 ....

For Maximum Wavelength

$\sin\theta = 1, n = 1 \Rightarrow \lambda_{max} = 2d$

so if $\lambda > 2d$ diffraction is not possible i.e. solution of Bragg's equation is not possible.

KEY POINTS

- Binding energy = – [Total Mechanical Energy]
- Velocity of electron in n^{th} orbit for hydrogen atom $\cong \frac{c}{137n}$; c = speed of light.
- Series limit means minimum wave length of that series.

NUCLEAR COLLISIONS

We can represent a nuclear collision or reaction by the following notation, which means X (a,b) Y

$$\underset{\text{(bombarding particle)}}{a} + \underset{\text{(at rest)}}{X} \rightarrow Y + b$$

We can apply :

(i) Conservation of momentum (ii) Conservation of charge (iii) Conservation of mass–energy

For any nuclear reaction $\underset{K_1}{a} + \underset{K_2}{X} \rightarrow \underset{K_3}{Y} + \underset{K_4}{b}$

By mass energy conservation

(i) $K_1 + K_2 + (m_a + m_x)c^2 = K_3 + K_4 + (m_Y + m_b)c^2$

(ii) Energy released in any nuclear reaction or collision is called Q value of the reaction.

(iii) $Q = (K_3 + K_4) - (K_1 + K_2) = \Sigma K_P - \Sigma K_R = (\Sigma m_R - \Sigma m_P)c^2$

(iv) If Q is positive, energy is released and products are more stable in comparison to reactants.

(v) If Q is negative, energy is absorbed and products are less stable in comparison to reactants.

$$Q = \Sigma(B.E.)_{product} - \Sigma(B.E.)_{reactants}$$

Nuclear Fission

In 1938 by Hahn and Strassmann. By attack of a particle splitting of a heavy nucleus (A > 230) into two or more lighter nuclei. In this process certain mass disappears which is obtained in the form of energy (enormous amount)

$$A + p \longrightarrow \text{excited nucleus} \longrightarrow B + C + Q$$

Nuclear Fusion :

It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

$$A + B \rightarrow C + Q \text{ (Energy)}$$

The product (C) is more stable then reactants (A and B) and $m_c < (m_a + m_b)$

and mass defect $\Delta m = [(m_a + m_b) - m_c]$ amu

Energy released is $E = \Delta m$ 931 MeV

The total binding energy and binding energy per nucleon C both are more than of A and B. $\Delta E = E_c - (E_a + E_b)$

RADIOACTIVITY

- **Radioactive Decays :** Generally, there are three types of radioactive decays (i) α decay (ii) β^- and β^+ decay (iii) γ decay
- **α decay:** By emitting α particle, the nucleus decreases it's mass number and move towards stability. Nucleus having A>210 shows α decay.
- **β decay** : In beta decay, either a neutron is converted into proton or proton is converted into neutron.
- **γ decay** : When an α or β decay takes place, the daughter nucleus is usually in higher energy state, such a nucleus comes to ground state by emitting a photon or photons.

  Order of energy of γ photon is 100 keV
- **Laws of Radioactive Decay :** The rate of disintegration is directly proportional to the number of radioactive atoms present at that time i.e., rate of decay $\propto$ number of nuclei.

  Rate of decay = λ (number of nuclei) i.e., $\frac{dN}{dt} = -\lambda N$

  where λ is called the decay constant.

  This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$.

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \Rightarrow \ell n\left(\frac{N}{N_0}\right) = -\lambda t$$

where N_0 is the number of parent nuclei at t=0. The number that survives at time t is therefore $N = N_0 e^{-\lambda t}$ and $t = \frac{2.303}{\lambda} \log_{10}\left(\frac{N_0}{N_t}\right)$

$N = N_0 e^{-\lambda t}$ where λ = decay constant

☐ Half life $t_{1/2} = \frac{\ell n 2}{\lambda}$

☐ Average life $t_{av} = \frac{1}{\lambda}$

- Within duration $t_{1/2} \Rightarrow$ 50% of N_0 decayed and 50% of N_0 remains active
- Within duration $t_{av} \Rightarrow$ 63% of N_0 decayed and 37% of N_0 remains active

☐ Activity $R = \lambda N = R_0 e^{-\lambda t}$

☐ 1Bq = 1 decay/s,

☐ 1 curie = 3.7×10^{10} Bq,

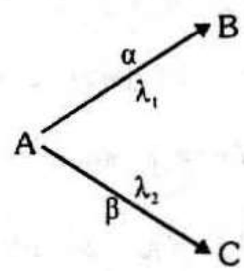
☐ 1 rutherford = 10^6Bq

☐ After n half lives Number of nuclei left $= \frac{N_0}{2^n}$

☐ Probability of a nucleus for survival of time t $= \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$

- **Parallel radioactive disintegration**

Let initial number of nuclei of A is N_0 then at any time number of nuclei of A, B & C are given by $N_0 = N_A + N_B + N_C$



$$\Rightarrow \frac{dN_A}{dt} = \frac{-d}{dt}(N_B + N_C)$$

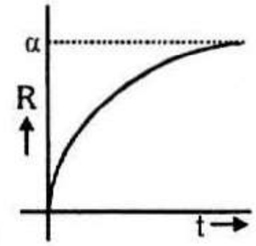
A disintegrates into B and C by emitting α,β particle.

$$\text{Now, } \frac{dN_B}{dt} = -\lambda_1 N_A \text{ and } \frac{dN_C}{dt} = -\lambda_2 N_A \Rightarrow \frac{d}{dt}(N_B + N_C) = -(\lambda_1 + \lambda_2) N_A$$

$$\Rightarrow \frac{dN_A}{dt} = -(\lambda_1 + \lambda_2) N_A \Rightarrow \lambda_{eff} = \lambda_1 + \lambda_2 \Rightarrow t_{eff} = \frac{t_1 t_2}{t_1 + t_2}$$

Radioactive Disintegration with Successive

Production $\xrightarrow[(\alpha = \text{rate of production})]{\alpha} A \xrightarrow{\lambda} B$



$$\frac{dN_A}{dt} = \alpha - \lambda N_A \text{ ....(i)}$$

when N_A is maximum $\frac{dN_A}{dt} = 0 \Rightarrow \alpha - \lambda N_A = 0$,

$$N_{A\,max} = \frac{\alpha}{\lambda} = \frac{\text{rate of production}}{\lambda}$$

By equation (i) $\int_0^t \frac{dN_A}{\alpha - \lambda N_A} = \int_0^t dt$, Number of nuclei is $N_A = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$

- **Equivalence of mass and energy** $E = mc^2$

Note :- $1u = 1.66 \times 10^{-27}$ kg $\equiv 931.5$ MeV or $c^2 = 931.5$ MeV/u

- **Binding energy of $_ZX^A$**

$BE = \Delta mc^2 = [Zm_p + (A-Z)m_n - m_X]c^2 = [Zm_H + (A-Z)m_n - m_X]c^2$

- **Q-value of a nuclear reaction**

For $a + X \longrightarrow Y + b$ or $X(a, b)Y$; $Q = (M_a + M_X - M_Y - M_b)c^2$

- **Radius of the nucleus**

$R = R_0A^{1/3}$ where $R_0 = 1.3\ f_m = 1.3 \times 10^{-15}$ m

From Bohr Model

$n_1 = 1$, $n_2 = 2, 3, 4$.......K series

$n_1 = 2$, $n_2 = 3, 4, 5$.......L series

$n_1 = 3$, $n_2 = 4, 5, 6$.......M series

OMPORTANT NOTES