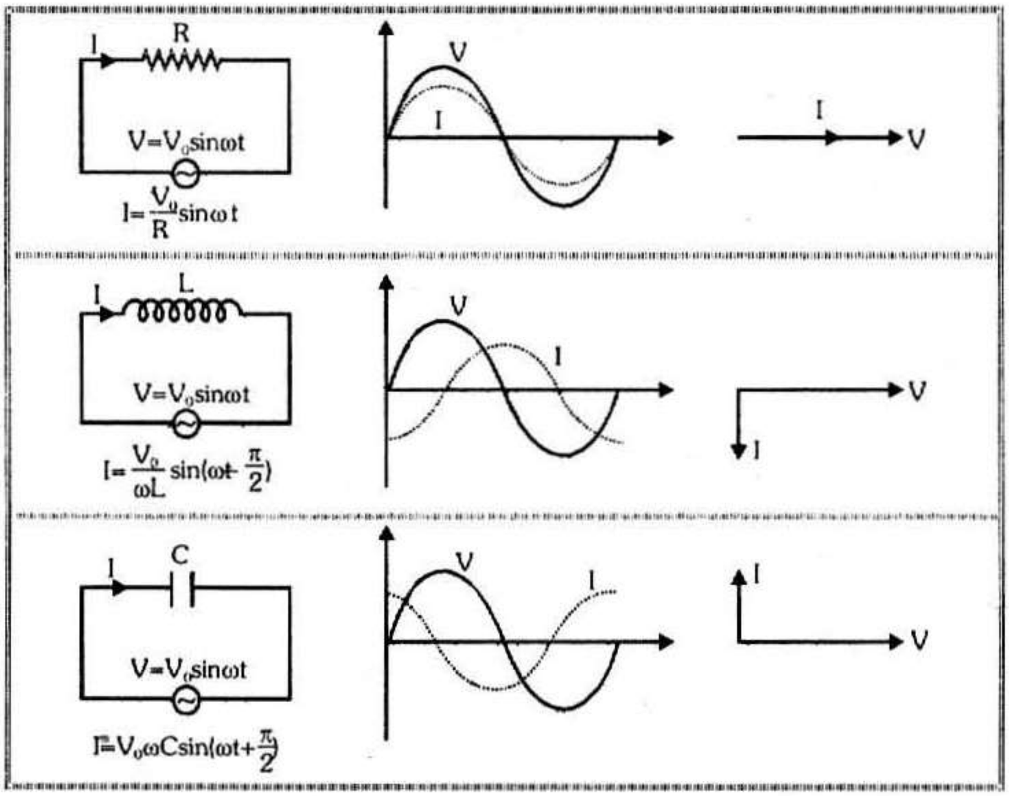
AC

- **Average value** $I_{av} = \dfrac{\int_0^T I\,dt}{\int_0^T dt} = \dfrac{1}{T}\int_0^T I\,dt$ • **RMS value** $I_{rms} = \sqrt{\dfrac{\int_0^T I^2\,dt}{\int_0^T dt}}$

- For sinusoidal voltage $V = V_0 \sin\omega t$: $V_{av} = \dfrac{2V_0}{\pi}$ & $V_{rms} = \dfrac{V_0}{\sqrt{2}}$

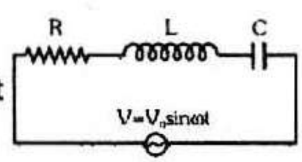
  For sinusoidal current $I = I_0 \sin(\omega t+\phi)$: $I_{av} = \dfrac{2I_0}{\pi}$ & $I_{rms} = \dfrac{I_0}{\sqrt{2}}$

- **AC Circuits**

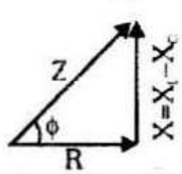


- **Impedance :** $Z = \sqrt{R^2 + X^2}$ where X = reactance

- **Series LCR Circuit**

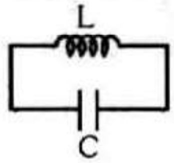


$Z = \sqrt{R^2 + (X_L - X_C)^2}$



$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

- **Power Factor** = $\cos\phi = R/Z$ At resonance : $X_L = X_C \Rightarrow Z = R, V = V_R$

- **LC Oscillation** 

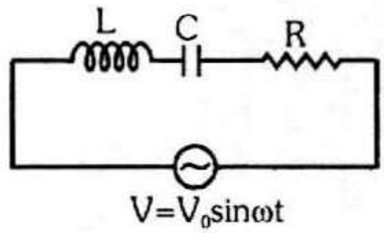
$q = q_0 \sin(\omega t + \theta)$, $I = I_0 \cos(\omega t + \theta)$ $\boxed{I_0 = q_0\omega}$

Energy = $\frac{1}{2}LI^2 + \frac{q^2}{2C} = \frac{q_0^2}{2C} = \frac{1}{2}LI_0^2$ = constant

Comparison with SHM $q \rightarrow x$, $I \rightarrow v$, $L \rightarrow m$, $C \rightarrow \frac{1}{K}$

Comparison of Damped Mechanical & electrical systems

- **(I) Series LCR circuit :**



$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = \frac{V_0}{L}\cos\omega t$$

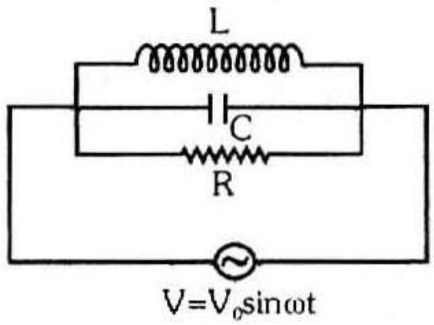
compare with mechanical damped system equation

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega t$$

where b = damping coefficient.

Mechanical system	Electrical systems (series RLC)
Displacement (x)	Charge (q)
Driving force (F)	Driving voltage (V)
Kinetic energy $\left(\frac{1}{2}mv^2\right)$	Electromagnetic energy of moving charge $\frac{1}{2}L\left(\frac{dq}{dt}\right)^2 = \frac{1}{2}Li^2$
Potential energy $\frac{1}{2}kx^2$	Energy of static charge $\frac{q^2}{2C}$
mass (m)	L
Power P =Fv	Power P=VI
Damping (b)	Resistance (R)
Spring constant	1/C

- **(II) Parallel LCR circuit :** In this case



$$I = I_L + I_C + I_R = \frac{\phi}{L} + \frac{d}{dt}C\left(\frac{d\phi}{dt}\right) + \frac{1}{R}\frac{d\phi}{dt} \Rightarrow \frac{d^2\phi}{dt^2} + \frac{1}{RC}\frac{d\phi}{dt} + \frac{1}{LC}\phi = \frac{V_0}{ZC}\sin\omega t$$

Displacement **(x)**	$\Longleftrightarrow$	Flux linkage **(ϕ)**
Velocity $\left(\frac{dx}{dt}\right)$	$\Longleftrightarrow$	Voltage $\left(\frac{d\phi}{dt}\right)$
Mass **(m)**	$\Longleftrightarrow$	Capacitance **(C)**
Spring constant **(k)**	$\Longleftrightarrow$	Reciprocal Industance **(1/L)**
Damping coefficient **(b)**	$\Longleftrightarrow$	Reciprocal resistance **(1/R)**
Driving force **(F)**	$\Longleftrightarrow$	Current **(*i*)**

- **Properties of EM Waves**
  - The electric and magnetic fields $\vec{E}$ and $\vec{B}$ are always perpendicular to the direction in which the wave is travelling. Thus the em wave is a transverse wave.
  - EM waves carry momentum and energy.
  - EM wave travel through vacuum with the speed of light c, where

    $$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$$

  - The instantaneous magnitude of $\vec{E}$ and $\vec{B}$ in an EM wave are related by the expression $\frac{E}{B} = c$
  - The cross product $\vec{E} \times \vec{B}$ always gives the direction in which the wave travels.
- **Poynting Vector** : The rate of flow of energy crossing a unit area by electromagnetic radiation is given by poynting vector $\vec{S}$ where $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$
- **Displacement current :** In a region of space in which there is a changing electric field, there is a displacement current defined as $I_d = \epsilon_0 \frac{d\phi_E}{dt}$ where ε_0 is the permittivity of free space and $\phi_E = \int \vec{E} \cdot d\vec{S}$ is the electric flux.

- **Maxwell's Equations**

$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$ [Gauss law for electricity]

$\oint \vec{B} \cdot d\vec{S} = 0$ [Gauss law for magnetism]

$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_B}{dt}$ [Faraday's law]

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left[I_C + \epsilon_0 \frac{d\phi_E}{dt} \right]$ [Ampere's law with Maxwell's correction]

KEY POINTS

- An alternating current of frequency 50 Hz becomes zero, 100 times in one second because alternating current changes direction and becomes zero twice in a cycle.
- An alternating current cannot be used to conduct electrolysis because the ions due to their inertia, cannot follow the changing electric field.
- Average value of AC is always defined over half cycle because average value of AC over a complete cycle is always zero.
- AC current flows on the periphery of wire instead of flowing through total volume of wire. This known as skin effect.

IMPORTANT NOTES