

— ATOMIC STRUCTURE —

JEE AND NEET Syllabus

1. *Introduction*
2. *Model*
3. *Some important Definitions & Their Meaning*
4. *Type of Line spectrum*
5. *Time Period and Frequency of Electron's Motion*
6. *Determination of No. of spectral lines (theoretical) in emission & in Absorption Transitions*
7. *Explanation of H-Spectrum & Spectral Line formula*
8. *General points For Spectral lines in every spectral Series*
9. *Concept of Reduced Mass & its Application in Bohr Theory*
10. *Concept of Recoiling of An Atom & Determination of Momentum & energy for Recoil Atoms*
11. *Shortcoming's of Bohr's Atomic model*
12. *Important Formulae*

KEY CONCEPT

1. Introduction

Write from the beginning of civilisation, people were very curious to know that what from the substances are made up of but, first significant attempt in this direction was made by Dalton & from where Dalton's atomic model starts.

Before we go into detail of any model, here first we must understand the difference between a model & a theory.

2. Model

A model is simply a set of hypothesis based on logical & scientific facts.

Theory : When any model satisfies majority of scientific queries by experimental verification then it is termed as theory otherwise, model is simply not accepted.

In Nutshell we can say that every theory is a model but every model is not a theory. So, after more & more clarity about the substances, various new models like Dalton, Thomson, Rutherford, Bohr etc came into the pictures.

2.1 Dalton's atomic model -

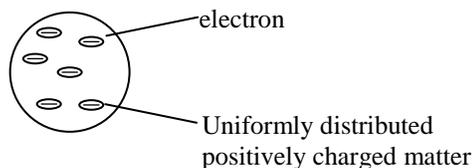
- (i) Every element is made up of tiny indivisible particles called atoms.
- (ii) Atoms of same element are identical both in physical & chemical properties while atoms of different elements are different in their properties
- (iii) All elements are made up of hydrogen atom. The mass of heaviest atom is about 250 times the mass of hydrogen atom while radius of heaviest atom is about 10 times the radius of hydrogen atom
- (iv) Atom is stable & electrically neutral.

Reason of Failure of model -

After the discovery of electron by J.J.Thomson (1897), it was established that atom can also be divide. Hence the model was not accepted.

2.2 Thomson's atomic model (or Plum-pudding model)

- (i) Atom is a positively charged solid sphere of radius of the order of 10^{-10}m in which electrons are embedded as seeds in a watermelon
- (ii) Total charge in a atom is zero & so, atom is electrically neutral



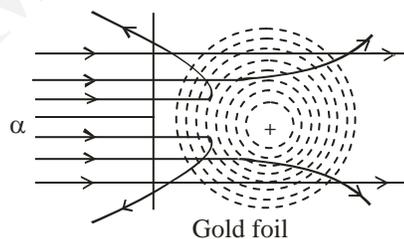
Achievements of model -

Explained successfully the phenomenon of thermionic emission, photoelectric emission & ionization

Failure of the model -

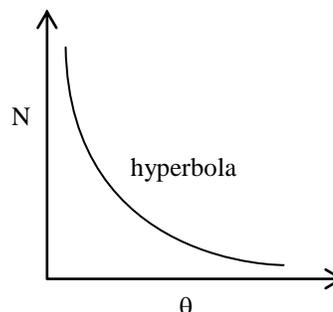
- (i) It could not explain the line spectrum of H-atom
- (ii) If could not explain the Rutherford's α -particle scattering experiment.

2.3 Rutherford's Atomic model -



Result of the experiment :

- (1) Majority of α -particles passes undeviated through gold foil that is possible when most part of the atom remains hollow.
- (2) Few of α -particles (< 1 in 8000) deflects at an angle larger than 90° & even some at 180° which is possible only in that case when there exists a solid positive mass confining in a very narrow space.
- (3)
$$N \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)} \Rightarrow \text{If } \theta \uparrow \text{ then } N \downarrow \downarrow .$$



Equation indicates that at larger deflection (scattering) angle, no. of particles deflected are very-very less



Graph for N & θ show that coulomb's law holds for atomic distances also

Rutherford's atomic model -

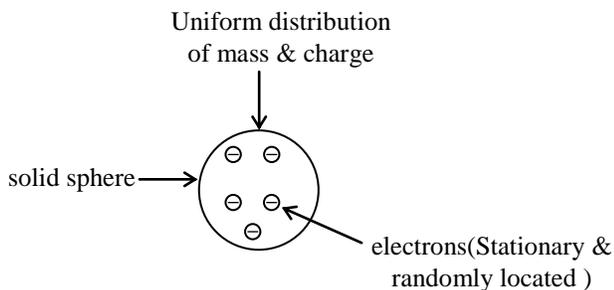


Fig : Thomson's model

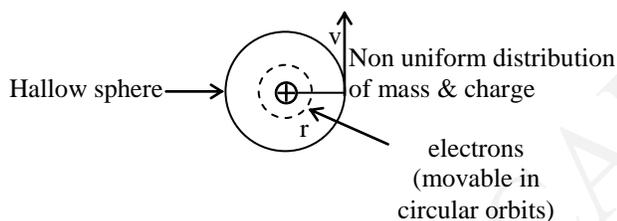
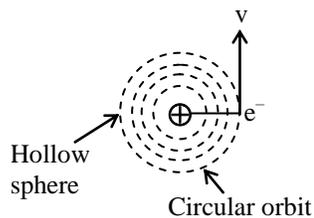


Fig : Rutherford's model

- (1) The whole positive charge & almost whole mass of an atom (leaving aside the mass of revolving e^- in various circular orbits) remains concentrated in a nucleus of radius of the order of $10^{-15}m$.



- (2) Σq (+)ve on proton in a nucleus = Σq (-)ve on e^- in various circular orbits & hence, the atom is electrically neutral.
- (3) The necessary centripetal force for revolving round the nucleus in circular orbit is provided

by coulomb's electrostatic force of attraction

$$\frac{mv^2}{r} = \frac{k(ze)(e)}{r^2}$$

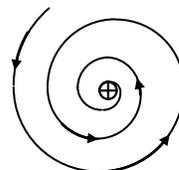
Reason of failure of model -

- (1) It could not explain the line spectrum of H-atom.

Justification : According to Maxwell's electromagnetic theory every accelerated moving charged particle radiates energy in the form of electromagnetic waves & therefore during revolution of e^- in circular orbit its frequency will continuously vary (i.e. decrease) which will result in the continuous emission of lines & therefore spectrum of atom must be continuous but in reality, one obtains line spectrum for atoms.

- (2) It could not explain the stability of atoms.

Justification - Since revolving electron will continuously radiates energy & therefore radii of circular path will continuously decrease & in a time of about 10^{-8} sec revolving electron must fall down in a nucleus by adopting a spiral path



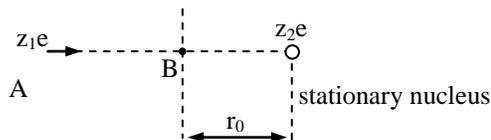
2.3.1 Application of rutherford's model

Determination of distance of closest approach :

When a positively charged particle approaches towards stationary nucleus then due to repulsion between the two, the kinetic energy of positively charged particle gradually decreases and a stage comes when its kinetic energy becomes zero & from where it again starts retracing its original path.

Definition : The distance of closest approach is the minimum distance of a stationary nucleus with a positively charged particle making head on collision from a point where its kinetic energy becomes zero.

Suppose a positively charged particle A of charge q_1 ($=z_1e$) approaches from infinity towards a stationary nucleus of charge z_2e then,



Let at point B, kinetic energy of particle A becomes zero then by the law of conservation of energy at point A & B,

$$TE_A = TE_B$$

$$KE_A + PE_A = KE_B + PE_B$$

$$E + 0 = 0 + \frac{k(z_1e)(z_2e)}{r_0} \quad (\text{in joule})$$

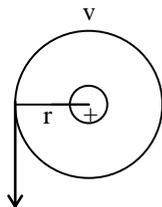
$$\therefore r_0 = \frac{k(z_1e)(z_2e)}{E} m$$

2.4 Bohr's Atomic model

Bohr proposed his model for H or H-like atoms by mixing the concepts of classical physics with quantum mechanics. This model is based on law of conservation of angular momentum.

- (1) (a) **Concept of stable, stationary, quantized, fixed allowed radii orbit, or maxwell's licensed orbits -**

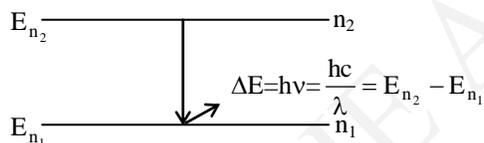
According to Bohr, if an electron revolve in these orbits then electron neither radiate nor absorb any energy.



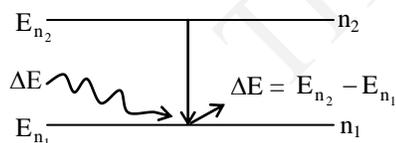
(b) Emission of energy

Where n = principle quantum no.

E_n = energy of e^- in n th orbit



(c) Absorption of energy



- (2) Electron revolve only in those orbits in which its angular momentum is integer multiple of $\frac{h}{2\pi}$

$$mvr = I\omega = n \frac{h}{2\pi}$$

$$(3) \frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

2.4.1 Determination of radius , velocity & energy of e^- in Bohr's orbit

(A) Determination of radius of circular path (orbit)

$$\therefore mvr = \frac{nh}{2\pi} \quad \dots(1)$$

$$\therefore v = \frac{nh}{2\pi mr} \quad \dots(2)$$

$$\& \frac{mv^2}{r} = \frac{kze^2}{r^2}$$

$$\therefore m \left(\frac{nh}{2\pi mr} \right) = \frac{kze^2}{r} ; r_n = v \left(\frac{n^2 h^2}{4\pi^2 m k z e} \right)$$

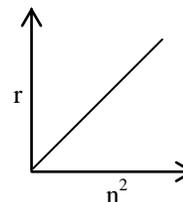
$$r_n = \frac{n^2}{z} \times \frac{h^2}{4\pi^2 m k e^2} ;$$

$$r_n = \frac{n^2}{z} \times 0.529 \text{ \AA} \quad \dots (3)$$

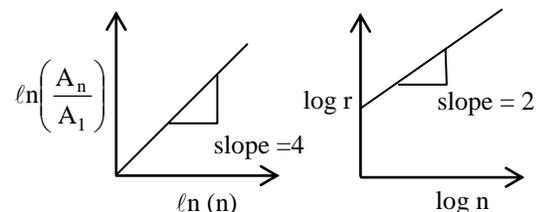
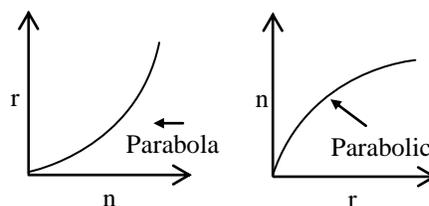
Results :

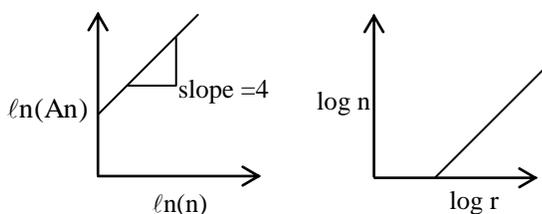
$$(i) \therefore r_1 = \frac{(1)^2}{z} \times 0.529 \text{ \AA}$$

$$\therefore r_n = n^2 r_1.$$



$$(ii) \therefore r \propto n^2$$





where $A_n =$ Area of n^{th} circular orbit

(B) Determination of velocity of electron in Circular orbit →

$$\therefore mvr = \frac{nh}{2\pi} \quad \dots(1)$$

$$r = \frac{nh}{2\pi mv}$$

$$\Rightarrow \frac{mv^2}{r} = \frac{kze^2}{r^2} \Rightarrow v = \frac{2\pi kze^2}{nh}$$

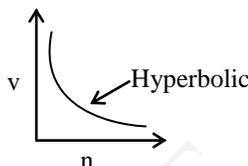
$$\Rightarrow v = \frac{z}{n} \times \frac{2\pi ke^2}{h} \Rightarrow v = 2.18 \times 10^6 \frac{z}{n} \text{ m/s}$$

$$v = \frac{c}{137} \frac{z}{n} \text{ m/s}$$

where $c =$ velocity of light in vacuum
 $= 3 \times 10^8 \text{ m/s}$

Results :

(i) $v \propto \frac{1}{n}$ ($z = \text{constant}$)

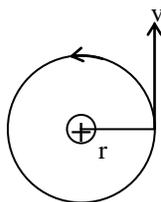


2.4.2 Determination of energy of electron in Bohr's circular orbit -

(1) Kinetic energy of electron

$$KE = \frac{1}{2} mv^2$$

$$KE = \frac{kze^2}{2r}$$



Results :

- (i) KE of an $e^- =$ positive quantity
- (ii) $r \uparrow$, $KE \downarrow$
- (iii) of $r = \infty$, $KE = 0$

(2) Potential energy of an electron

$$PE = \frac{K(+ze)(-e)}{r}$$

$$PE = - \frac{Kze^2}{r}$$

Results :

- (i) PE of an $e^- =$ negative quantity
- (ii) $r \uparrow$, $PE \uparrow$
- (iii) If $r = \infty$, $PE = 0$

(3) Total energy of an electron : Total energy of an electron in any orbit is sum of its kinetic & potential energy in that orbit.

$$TE = KE + PE = \frac{Kze^2}{2r} - \frac{Kze^2}{r} ; \quad TE = - \frac{Kze^2}{2r}$$

Results :

- (i) TE of an electron in an atom = (-)ve quantity. (-)ve sign indicates that electron is in bound state
- (ii) If $r \uparrow$, $TE \uparrow$
- (iii) if $r = \infty$, $TE = 0$
- (iv) $TE = - KE = \frac{PE}{2}$ in any H-like atom

Total energy in term of n

$$TE = - \frac{kze^2}{2 \times \left(\frac{n^2 h^2}{4\pi^2 m k z e^2} \right)}$$

$$\Rightarrow TE = - \frac{2\pi^2 m k^2 z^2 e^4}{n^2 h^2} \Rightarrow TE = - Rch \frac{z^2}{n^2} \text{ joule}$$

$$\Rightarrow TE = - 13.6 \frac{z^2}{n^2} \text{ eV}$$

where $R =$ Rydberg constant $= \frac{2\pi^2 m k^2 e^4}{ch^3}$

$$= \frac{me^4}{8\epsilon_0^2 ch^3} = 1.097 \times 10^7 \text{ m}^{-1}$$

* Remember that Rydberg constant is not a universal constant. In Bohr calculation it is determined by assuming nucleus to be stationary

For Bohr Rydberg constant

$$R_\infty = 1.097 \times 10^7 \text{ m}^{-1}$$

If nucleus is not assumed stationary then

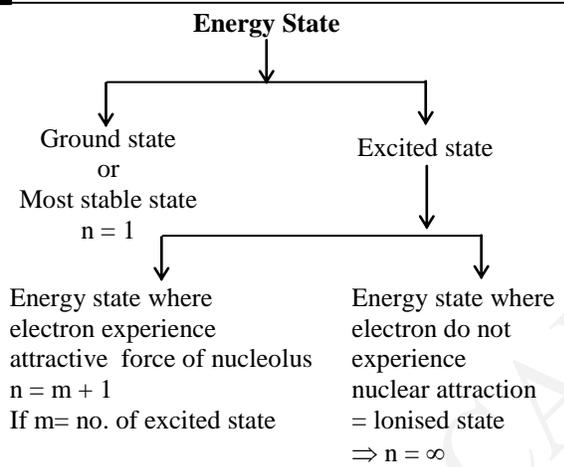
$$R = \frac{R_\infty}{1 + \left(\frac{m_e}{m_N} \right)}, \quad m_N = \text{mass of nucleus}$$

2.4.3 Results based on total energy equation

- (i) With the increase in principal quantum number, both total energy & potential energy of an electron increases. While kinetic energy decreases

- (ii) With the increase in principal quantum number, the difference between any two consecutive energy level decreases.
- (iii) Total energy of an electron in any orbit in H-like atom = (Total energy of an electron in that orbit in H-atom) $\times z^2$
- (iv) PE of an electron in any orbit in H-like atom = (PE of an electron in that orbit in H-atom) $\times z^2$
- (v) KE of an electron in any orbit in H-like atom = (KE of an electron in that orbit in H-atom) $\times z^2$
- (vi) $\Delta E_{n_1, n_2}$ in any H-like atom = ($\Delta E_{n_1, n_2}$ in H-atom) $\times z^2$

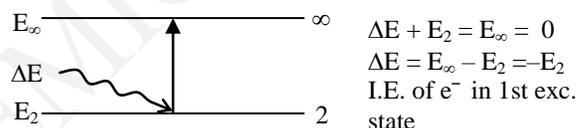
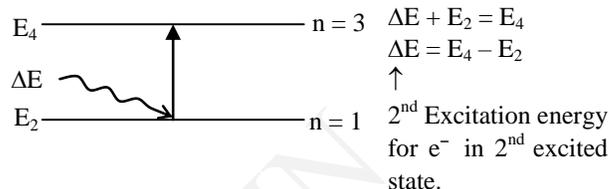
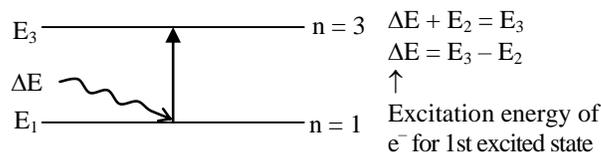
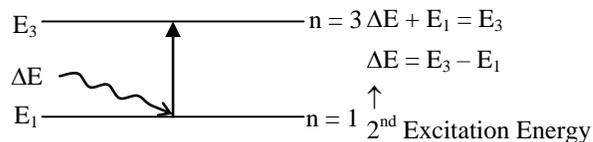
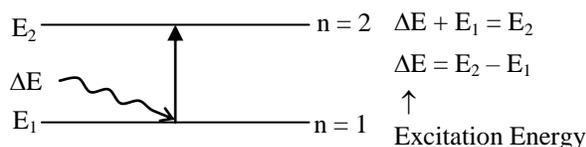
3. Some important definitions & their meaning



(1) **Ionization energy & ionization potential** - The minimum energy required to remove an electron from hydrogen or hydrogen like atom is called its ionization energy & corresponding potential through which an electron is accelerated for this is called ionization potential

$$\text{I.E.} = E_{\infty} - E_1 = -E_1 = \text{Binding energy of } e^-$$

(2) **Excitation Energy & excitation potential** - The minimum energy required to excite an atom is called excitation energy of the particular excited state & corresponding potential is called excitation potential.



If excitation energy & ionisation energy are represented in eV then corresponding value in volt is termed as excitation potential & ionisation potential respectively.

For example : Excitation energy & Ionisation energy for H-atom are 10.2 eV & 13.6 eV respectively & there fore 10.2V & 13.6V are excitation & ionisation potential respectively.

4. Type of line spectrum

Emission line spectrum :

When an atomic gas or vapour at a pressure less than the atmospheric pressure is excited by passing electric discharge, the emitted radiation has a spectrum which contains certain specific bright lines only. These emission lines constitute emission spectrum. These are obtained when electron jumps from excited states to lower states. The wavelength of emission lines of different elements are different. For one element the emission spectrum is unique. It is used for the determination of composition of an unknown substance.

Absorption line spectrum :

When white light is passed through a gas, the gas is found to absorb light of certain wavelength. The bright background on the photographic plate is then crossed by dark lines that corresponds to those wavelengths which are absorbed by the gas atoms.

The absorption spectrum consists of dark lines on bright background. These are obtained due to absorption of certain wavelengths, resulting into transition of atom from lower energy states to higher energy states. (The emission spectrum consists of bright lines on dark background.)

The spectrum of sunlight has dark lines called **Fraunhofer lines**. These are produced when the light coming out of the interior (core) of the sun, passes through the envelope of cooler gas. The cooler gas absorbs light of certain wavelengths corresponding to the elements present in the cooler gas. This results into dark lines (absence of wavelength) on brighter background. Study of Fraunhofer lines is used to determine the elements (composition) of the star.

5. Time Period and Frequency of Electron's Motion

Time period of revolution of an electron in the n^{th} Bohr orbit is

$$T_n = \frac{2\pi r_n}{v_n}$$
$$= \frac{n^3}{Z^2} \frac{h^3}{4\pi^2 m k^2 e^4}$$

For H-atom, $Z = 1$; then for $n = 1$,

$$T_1 = 1.5 \times 10^{-16} \text{ sec}$$

$$T_1 : T_2 : T_3 = 1 : 8 : 27$$

Frequency of revolution

$$v_n = \frac{1}{T_n}$$
$$v_n \propto \frac{Z^2}{n^3}$$

For H-atom $v_1 = 6.6 \times 10^{15}$ Hz,

$$v_1 : v_2 : v_3 = 1 : \frac{1}{8} : \frac{1}{27}$$

Current and Magnetic field Due to Electron's Motion

The motion of electron in circular orbit, give rise to some equivalent current in the orbit, it is equal to (in the n^{th} orbit).

$$I_n = ev_n$$
$$= \frac{Z^2}{n^3} \left(\frac{4\pi^2 m k^2 e^5}{h^3} \right)$$
$$I_n \propto \frac{Z^2}{n^3}$$

For H-atom, $I_1 = 1 \text{ mA}$

The magnetic field at the centre of the orbit, (at nucleus) is

$$B_n = \frac{\mu_0 I}{2a_0}$$
$$= \left(\frac{\mu_0 8\pi^4 m^2 k^3 e^7}{h^5} \right)$$

$$b_n \propto Z^3/n^5$$

For H-atom, $B_1 = 12.5 \text{ tesla}$

The magnetic moment

(orbital) due to electrons orbital motion is

$$M = \text{current} \times \text{area}$$

$$M_n = I_n \cdot \pi r_n^2$$

$$M_n = \frac{nhv}{4\pi m} ; \quad M_n = \frac{eL}{2m}$$

where $L = \frac{nh}{2\pi}$, angular momentum of the electron in

its orbit. The value of magnetic moment in first Bohr orbit is called Bohr magneton (μ_B). Its value is

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} \text{ Am}^2.$$

Comment In my view, you should not try to cram the formulas for T_n , v_n , I_n , B_n . Usually no one is going to ask the full form. What you must memorise is their dependence on Z and n and order of magnitudes in first Bohr orbit.

$$T_n \propto n^3/Z^2 ; \quad T_1 \approx 1.5 \times 10^{-16} \text{ sec}$$

$$v_n \propto Z^2/n^3 ; \quad v_1 \approx 6.6 \times 10^{15} \text{ Hz}$$

$$I_n \propto Z^2/n^3 ; \quad I_1 \approx 1 \text{ mA}$$

$$B_n \propto Z^3/n^5 ; \quad B_1 \approx 12.5 \text{ T}$$

$$M_n \propto n ; \quad M_1 = \mu_B$$
$$\approx 9.27 \times 10^{-24} \text{ Am}^2$$

$$\omega_n = 2\pi v_n ; \quad \omega_n \propto Z^2/n^3$$

$$L_n = nh/2\pi ; \quad L_n \propto n$$

6. Determination of no. of spectral lines (theoretical) in emission & in absorption transitions

6.1 No. of emission spectral lines - If the electron is excited to state with principal quantum number n then from the n^{th} state, the electron may go to $(n - 1)^{\text{th}}$ state,, 2^{nd} state or 1st state. So there are $(n - 1)$ possible transitions starting from the n^{th} state. The electron reaching $(n - 1)^{\text{th}}$ state may make $(n - 2)$ different transitions. Similarly for other lower states. The total no. of possible transitions is $(n - 1) + (n - 2) + (n - 3) + \dots + 2 + 1 = \frac{n(n - 1)}{2}$

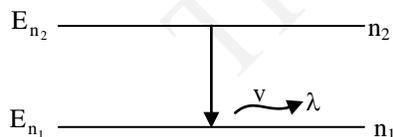
6.2 No. of absorption spectral lines - Since at ordinary temperatures, almost all the atoms remain in their lowest energy level ($n = 1$) & so absorption transition can start only from $n = 1$ level (not from $n = 2, 3, 4, \dots$ levels). Hence, only Lyman series is found in the absorption spectrum of hydrogen atom (which as in the emission spectrum, all the series are found)

No. of absorption spectral lines = $(n - 1)$

Remember : The absorption spectrum of sun has Balmer series also besides the Lyman series. Many H-atoms remain in $n = 2$ also due to very high temperature.

7. Explanation of H-spectrum & spectral line formula

In a hydrogen like atom, when an electron makes transition from any higher energy state n_2 to any lower energy state n_1 then a photon of frequency ν or wavelength λ is emitted.



$$\text{Then } \Delta E = h\nu = \frac{hc}{\lambda} = E_{n_2} - E_{n_1}$$

$$\therefore E = -Rch \frac{Z^2}{n^2} \quad J = -13.6 \frac{Z^2}{n^2} \text{ eV}$$

$$\therefore \Delta E = -\frac{RchZ^2}{n_2^2} - \left(-\frac{RchZ^2}{n_1^2} \right)$$

$$\Delta E = RchZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\therefore h\nu = RchZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\bar{\nu} = \frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$\bar{\nu}$ = wave number

= no. of wave in unit length

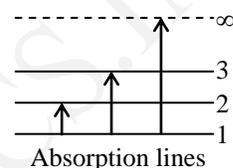
$$\nu = c \bar{\nu}$$

For H-atom, $Z = 1$ & therefore,

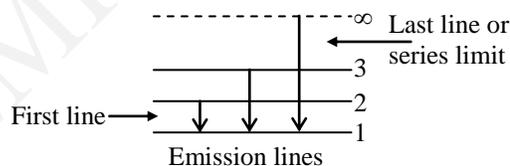
$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

(1) Lyman series -

$$n_1 = 1, n_2 = 2, 3, 4, \dots, \infty$$



Absorption lines



Emission lines

For 1st line or series beginning

$$n_1 = 1, n_2 = 2$$

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$\lambda_{\text{max}} = 1216 \text{ \AA}$$

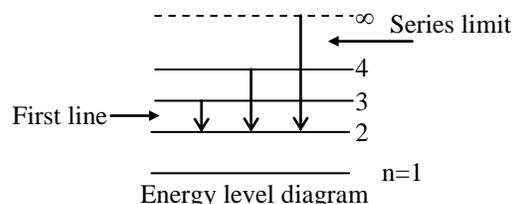
For series limit or last line $n_1 = 1, n_2 = \infty$

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{\infty^2} \right]$$

$$\lambda_{\text{min}} = \frac{1}{R} = 912.68 \text{ \AA}$$

* Remember - Lyman series is found in UV region of electromagnetic spectrum

(2) Balmer series -



Energy level diagram $n=1$

$$n_1 = 2, n_2 = 3, 4, 5, 6, \dots, \infty$$

wavelength of first line

i.e. maximum wavelength $\frac{1}{\lambda_{\max}} = R \left[\frac{1}{2^2} - \frac{1}{3^2} \right]$

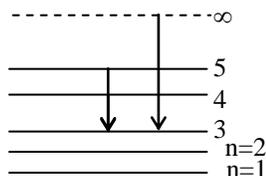
$\therefore \lambda_{\max} = 6563 \text{ \AA}$

wavelength of last line or series limit i.e. minimum wavelength

$\lambda_{\min} = R \left[\frac{1}{2^2} - \frac{1}{\infty^2} \right]; \lambda_{\min} = \frac{4}{R} = 3646 \text{ \AA}$

- * Balmer series is found only in emission spectrum
- * Balmer series lies in the visible region of electromagnetic spectrum

(3) Paschen series -



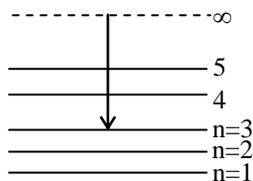
$n_1 = 3, n_2 = 4, 5, 6, \dots$

For first line $n_1 = 3, n_2 = 4$

$\frac{1}{\lambda_{\max}} = R \left[\frac{1}{3^2} - \frac{1}{4^2} \right]$

$\lambda_{\max} = 18751 \text{ \AA}$

For last line or series limit



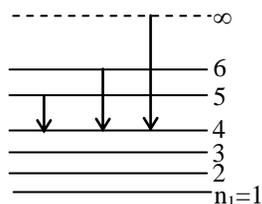
$n_1 = 3, n_2 = \infty$

$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{3^2} - \frac{1}{\infty^2} \right]$

$\lambda_{\min} = \frac{9}{R} = 8107 \text{ \AA}$

- * Paschen series is also found only in emission spectrum
- * Paschen series is obtained in infrared region of electromagnetic spectrum

(4) Brakett series -



$n_1 = 4, n_2 = 5, 6, 7, \dots \infty$

For first line $\frac{1}{\lambda_{\max}} = R \left[\frac{1}{4^2} - \frac{1}{5^2} \right]$

$\lambda_{\max} = 40477 \text{ \AA}$

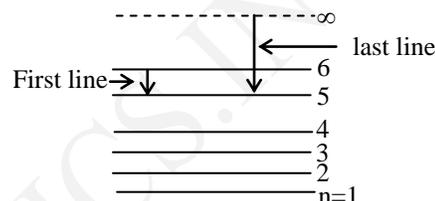
For last line or series limit

$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{4^2} - \frac{1}{\infty^2} \right]$

$\lambda_{\min} = \frac{16}{R} = 14572 \text{ \AA}$

- * Brakett series is also found only in emission spectrum
- * Brakett series is also obtained in infrared region of electromagnetic spectrum

(5) Pfund series -



$n_1 = 5, n_2 = 6, 7, 8, \dots \infty$

For first line $\frac{1}{\lambda_{\max}} = R \left[\frac{1}{5^2} - \frac{1}{6^2} \right]$

$\lambda_{\max} = 74515 \text{ \AA}$

For last line or series limit

$\frac{1}{\lambda_{\min}} = R \left[\frac{1}{5^2} - \frac{1}{\infty^2} \right]$

$\lambda_{\min} = \frac{25}{R} = 22768 \text{ \AA}$

- * Pfund series is also obtained only in emission spectrum
- * Pfund series is situated in the infrared region of electromagnetic spectrum

8. General points for spectral lines In every spectral series

- (1) Wavelength of first line is maximum & last line is minimum.
- (2) As the order of spectral series increases, wavelength also usually increases
 $\lambda_{PF} > \lambda_{BR} > \lambda_P > \lambda_B > \lambda_L$
- (3) Frequency of energy emission in lyman transitions are highest among all other series.

9. Concept of Reduced mass & its application in Bohr theory

- (i) When mass of nucleus is assumed to be very-very large in comparison to mass of revolving particle then reduced mass is not to be applied otherwise it is to be applied
- (ii) Bohr has assumed nucleus to be stationary in its calculations.
- (iii) Reduced mass of a system of particles of mass m_1 & m_2 is written by

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

- (iv) According to Bohr

$$\mu = \frac{m_e m_N}{m_e + m_N} = \frac{m_e m_N}{m_N}$$

$$\therefore m_N \gg m_e = m_e$$

where m_N = mass of nucleus

- (v) For Muon - Proton system

mass of muonic atom = $207 \times$ mass of electron (m_{μ^-})

$$\therefore \mu = \frac{m_p \times m_{\mu^-}}{m_p + m_{\mu^-}} = \frac{1836m_e \times 207m_e}{(1836 + 207)m_e}$$

$$\mu \approx 207 m_e$$

$$\mu \approx 186 m_e$$

- (vi) For electron - positron system

$$\mu = \frac{m_{e^-} \times m_{e^+}}{m_{e^-} + m_{e^+}}$$

$$\mu = \frac{m_e}{2}$$

m_e = mass of electron

- (vii) radius in n^{th} orbit

$$r_n = \frac{n^2}{z} \times 0.529 \times \frac{m_e}{\mu} \text{ \AA}$$

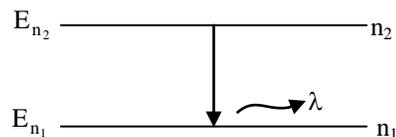
- (viii) Energy in n^{th} orbit

$$E_n = -\frac{z^2}{n^2} \times 13.6 \times \frac{\mu}{m_e} \text{ eV}$$

10. Concept of recoiling of an atom & determination of momentum & energy for recoil atoms

When ever any electron makes transition from any higher energy state to any lower energy state then photon is emitted & due to which in the back side

atom is recoiled. The atom is recoiled by sharing some energy from the energy evolved during electronic transition.



If m = mass of recoil atom

v = velocity of recoil atom

$$\text{Then } \frac{1}{2}mv^2 + \frac{hc}{\lambda} = E_{n_2} - E_{n_1} = \Delta E$$

Recoil momentum of atom = $\frac{h}{\lambda}$ = momentum of photon

$$\text{Recoil energy of atom} = \frac{p^2}{2m}$$

11. Shortcoming's of Bohr's Atomic model

- It is valid only for one electron atoms. e.g : H, He⁺, Li²⁺, Na⁺¹⁰ etc.
- Orbits were taken as circular but according to Sommerfeld these are elliptical.
- Intensity of spectral lines could not be explained.
- Nucleus was taken as stationary but it also rotates on its own axis.
- It could not be explained the minute structure in spectrum line.
- This does not explain the Zeeman effect (splitting up of spectral lines in magnetic field) & Stark effect (splitting up in electric field)
- This does not explain the doublets in the spectrum of some of the atoms like sodium (5890Å & 5896Å)

12. Important formulae

(1) Time period (T) :

distance = time \times speed

$$2\pi r = T \times v$$

$$T = \frac{2\pi r}{v}$$

$$T_n = \frac{n^2 h^3}{4\pi k^2 z^2 m_e^4}, T_n \propto \frac{n^3}{z^2}$$

(2) Frequency of revolution

$$f_n = \frac{v_n}{2\pi r_n} = \frac{4\pi k^2 z^2 m e^4}{n^3 h^3} ; f_n \propto \frac{z^2}{n^3}$$

(3) Momentum of electron

$$P_n = \frac{2\pi m k z^2}{nh} \quad P_n \propto \frac{1}{n}$$

(4) Angular velocity of electron

$$\omega_n = \frac{8\pi^3 k^2 z^2 m e^4}{n^3 h^3} , \quad \omega_n \propto \frac{z^2}{n^3}$$

(5) Current (I)

$$I = \frac{e}{T} = ev = \frac{ev}{2\pi r} ; \quad I \propto \frac{z^2}{n^3}$$

(6) Magnetic moment of electron (M)

$$M = iA$$

$$M = \frac{eV}{2\pi r} \times \pi r^2 , \quad M = \frac{evr}{2}$$

$$M = \frac{e(mvr)}{2m} = \frac{eJ}{2m} ; \quad \frac{M}{J} = \frac{e}{2m} ,$$

$$M = \frac{e}{2m} \left(\frac{nh}{2\pi} \right) = n \left(\frac{eh}{4\pi m} \right)$$

$$M = n\mu_B \quad \mu_B = \text{Bohr magneton} \\ = 9.3 \times 10^{-24} \text{ Amp.m}^2.$$

$$M \propto n$$

(7) Magnetic field or Magnetic induction at the centre

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 ev}{4\pi r^2}$$

- (8) 4 _____ - 0.85 eV
3 _____ - 1.51 eV
2 _____ - 3.4 eV
n = 1 _____ - 13.6 eV

Energy levels values for H-atom

(9) Difference of energy levels in H-atom

$$\Delta E_{12} = 10.2 \text{ eV} \quad \Delta E_{24} = 2.55 \text{ eV}$$

$$\Delta E_{13} = 12.09 \text{ eV} \quad \Delta E_{23} = 1.89 \text{ eV}$$

$$\Delta E_{14} = 12.75 \text{ eV}$$

$$(10) \Delta E = \frac{hc}{\lambda^2} \Delta\lambda$$

where λ = mean wavelength

$\Delta\lambda$ = difference in wavelength

ΔE = difference in energy levels

$$(11) \text{Total no. of electron in a shell} = 2n^2$$

$$(12) \text{Total no. electron in a subshell} = 2(2l + 1)$$