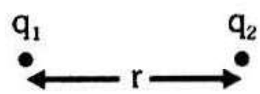
ELECTROSTATICS

Electric Charge

Charge of a material body is that property due to which it interacts with other charges. There are two kinds of charges- positive and negative. S.I. unit is coulomb. Charge is quantized, conserved, and additive.

Coulomb's law :

Force between two charges $\vec{F} = \dfrac{1}{4\pi \epsilon_0 \epsilon_r} \dfrac{q_1 q_2}{r^2} \hat{r}$ ϵ_r = dielectric constant



NOTE :The Law is applicable only for static and point charges. Moving charges may result in magnetic interaction. And if charges are extended, induction may change the charge distribution.

Principle Of Superposition

Force on a point charge due to many charges is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots\dots\dots$$

NOTE : The force due to one charge is not affected by the presence of other charges.

Electric Field or Electric Field Intensity or Electric Field Strength (Vector Quantity)

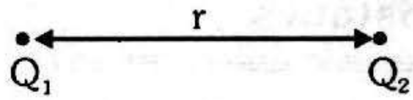
In the surrounding region of a charge there exist a physical property due to which other charge experiences a force. The direction of electric field is direction of force experienced by a positively charged particle and the magnitude of the field (electric field intensity) is the force experienced by a unit charge.

$$\vec{E} = \frac{\vec{F}}{q} \text{ unit is N/C or V/m.}$$

- **Electric field intensity due to charge Q**

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2} \hat{r}$$

Null point for two charges :



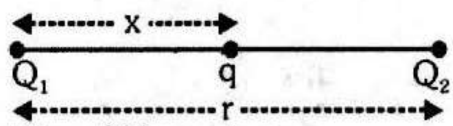
If $|Q_1| > |Q_2|$

$\Rightarrow$ Null point near Q_2

$$x = \frac{\sqrt{Q_1}\,r}{\sqrt{Q_1} \pm \sqrt{Q_2}}$$ $x \rightarrow$ distance of null point from Q_1 charge

(+) for like charges
(–) for unlike charges

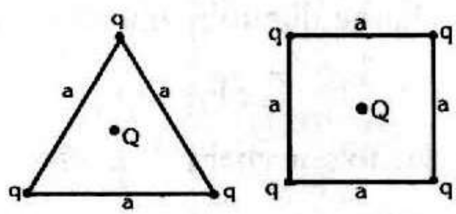
☐ Equilibrium of three point charges



(i) Two charges must be of like nature.
(ii) Third charge should be of unlike nature.

$$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}}\,r \quad \text{and} \quad q = \frac{-Q_1 Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$$

☐ Equilibrium of symmetric geometrical point charged system



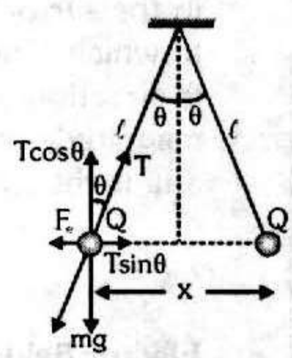
Value of Q at centre for which system to be in state of equilibrium

(i) For equilateral triangle $Q = \frac{-q}{\sqrt{3}}$ (ii) For square $Q = \frac{-q(2\sqrt{2} + 1)}{4}$

☐ Equilibrium of suspended point charge system

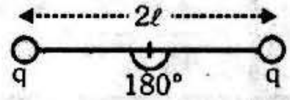
For equilibrium position

$$T\cos\theta = mg \;\&\; T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$



$$T = \sqrt{(F_e)^2 + (mg)^2}$$

- If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$)



$$T = F_e = \frac{kq^2}{4\ell^2}$$

- **Electric potential difference** $\Delta V = \dfrac{\text{work}}{\text{charge}} = W/q$

- **Electric potential** $V_P = -\int_{\infty}^{P} \vec{E}.\vec{dr}$

  It is the work done against the field to take a unit positive charge from infinity (reference point) to the given point

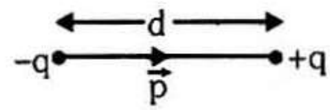
  ☐ For point charge : $V = K\dfrac{q}{r}$ ☐ For several point charges : $V = K\sum\dfrac{q_i}{r_i}$

- **Relation between $\vec{E}$ & V**

  $\vec{E} = -\text{grad } V = -\nabla V$, $E = -\dfrac{\partial V}{\partial r}$; $\vec{E} = -\dfrac{\partial V}{\partial x}\hat{i} - \dfrac{\partial V}{\partial y}\hat{j} - \dfrac{\partial V}{\partial z}\hat{k}$, $V = \int -\vec{E}.\vec{dr}$

- **Electric potential energy of two charges** : $U = \dfrac{1}{4\pi \epsilon_0}\dfrac{q_1 q_2}{r}$

- **Electric dipole**



  ☐ Electric dipole moment $p = qd$

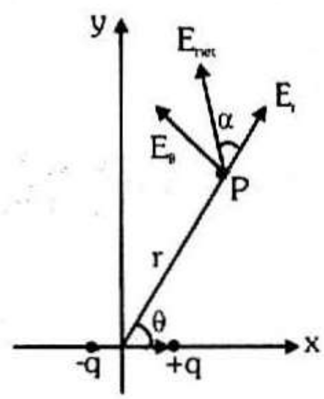
  ☐ Torque on dipole placed in uniform electric field $\vec{\tau} = \vec{p} \times \vec{E}$

  ☐ Work done in rotating dipole placed in uniform electric field

  $$W = \int \tau d\theta = \int_{\theta_0}^{\theta} PE \sin\theta d\theta = pE(\cos\theta_0 - \cos\theta)$$

  ☐ Potential energy of dipole placed in an uniform field $U = -\vec{p}.\vec{E}$

  ☐ At a point which is at a distance r from dipole midpoint and making angle θ with dipole axis.



  • Potential $V = \dfrac{1}{4\pi \epsilon_0}\dfrac{p\cos\theta}{r^2}$

  • Electric field $E = \dfrac{1}{4\pi \epsilon_0}\dfrac{p\sqrt{1+3\cos^2\theta}}{r^3}$

  • Direction $\tan\alpha = \dfrac{E_\theta}{E_r} = \dfrac{1}{2}\tan\theta$

  ☐ Electric field at axial point (or End-on) $\vec{E} = \dfrac{1}{4\pi \epsilon_0}\dfrac{2\vec{p}}{r^3}$ of dipole

  ☐ Electric field at equatorial position (Broad-on) of dipole $\vec{E} = \dfrac{1}{4\pi \epsilon_0}\dfrac{(-\vec{p})}{r^3}$

Equipotential Surface And Equipotential Region

In an electric field the locus of points of equal potential is called an equipotential surface. An equipotential surface and the electric field line meet at right angles. The region where $E = 0$, Potential of the whole region must remain constant as no work is done in displacement of charge in it. It is called as equipotential region like conducting bodies.

Mutual Potential Energy Or Interaction Energy

"The work to be done to integrate the charge system".

For 2 particle system $U_{mutual} = \dfrac{q_1q_2}{4\pi\varepsilon_0 r}$

For 3 particle system $U_{mutual} = \dfrac{q_1q_2}{4\pi\varepsilon_0 r_{12}} + \dfrac{q_2q_3}{4\pi\varepsilon_0 r_{23}} + \dfrac{q_3q_1}{4\pi\varepsilon_0 r_{31}}$

For n particles there will be $\dfrac{n(n-1)}{2}$ terms .

Total energy of a system = $U_{self} + U_{mutual}$

Electric flux : $\phi = \int \vec{E}.d\vec{s}$

(i) For uniform electric field; $\phi = \vec{E}.\vec{A} = EA\cos\theta$ where θ = angle between $\vec{E}$ & area vector $(\vec{A})$. Flux is contributed only due to the component of electric field which is perpendicular to the plane.

(ii) If $\vec{E}$ is not uniform throughout the area A , then $\phi = \int \vec{E}.d\vec{A}$

Gauss's Law : $\oint \vec{E}.d\vec{s} = \dfrac{\sum q}{\epsilon_0}$ (Applicable only to closed surface)

Net flux emerging out of a closed surface is $\dfrac{q_{en}}{\varepsilon_0}$.

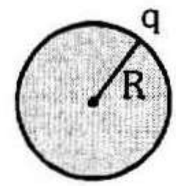
$\phi = \oint \vec{E}\cdot d\vec{A} = \dfrac{q_{en}}{\varepsilon_0}$ where q_{en}= net charge enclosed by the closed surface .

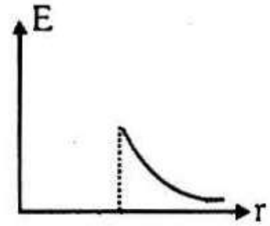
ϕ does not depend on the

(i) Shape and size of the closed surface

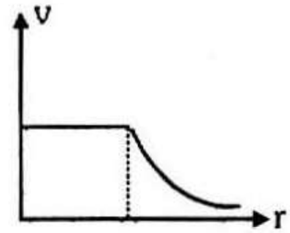
(ii) The charges located outside the closed surface.

- **For a conducting sphere**



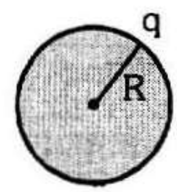


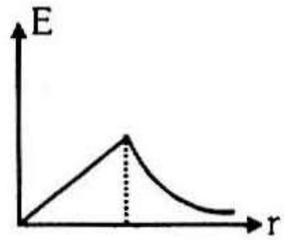
- ☐ For $r \geq R : E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2}$, $V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}$



- ☐ For $r < R$: $E = 0$, $V = \frac{1}{4\pi \epsilon_0}\frac{q}{R}$

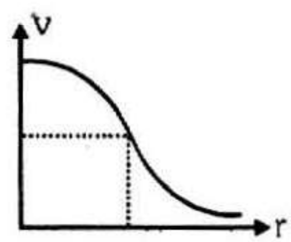
- **For a non-conducting sphere**





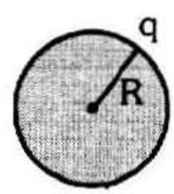
- ☐ For $r \geq R : E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2}$, $V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}$

- ☐ For $r < R : E = \frac{1}{4\pi \epsilon_0}\frac{qr}{R^3}$, $V = \frac{1}{4\pi \epsilon_0}\frac{q(3R^2 - r^2)}{2R^3}$



$$V_C = V_{max} = \frac{3}{2}\frac{Kq}{R} = 1.5V_{Surface}$$

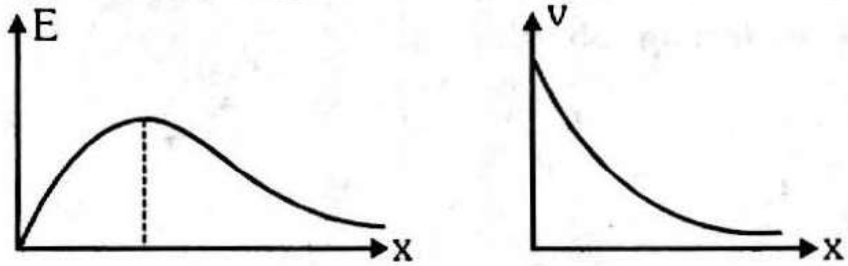
- **For a conducting/non conducting spherical shell**

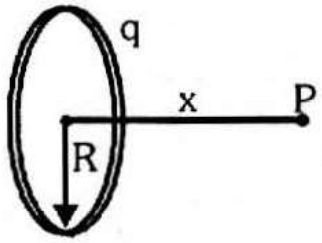


- ☐ For $r \geq R : E = \frac{1}{4\pi \epsilon_0}\frac{q}{r^2}$, $V = \frac{1}{4\pi \epsilon_0}\frac{q}{r}$

- ☐ For $r < R : E = 0$, $V = \frac{1}{4\pi \epsilon_0}\frac{q}{R}$

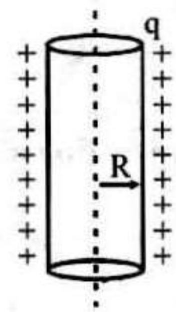
- **For a charged circular ring**





$$E_P = \frac{1}{4\pi \epsilon_0}\frac{qx}{(x^2+R^2)^{3/2}},\ V_P = \frac{1}{4\pi \epsilon_0}\frac{q}{(x^2+R^2)^{1/2}}$$

Electric field will be maximum at $x = \pm\frac{R}{\sqrt{2}}$



- **For a charged long conducting cylinder**

  - For $r \geq R : E = \frac{q}{2\pi \epsilon_0 r}$

  - For $r < R : E = 0$

- **Electric field intensity at a point near a charged conductor** $E = \frac{\sigma}{\epsilon_0}$

- **Mechanical pressure on a charged conductor**

  $P = \frac{\sigma^2}{2\epsilon_0}$

- **For non-conducting long sheet of surface charge density σ** $E = \frac{\sigma}{2\epsilon_0}$

- **For conducting long sheet of surface charge density** $E = \frac{\sigma}{\epsilon_0}$

- **Energy density in electric field** $u_E = \frac{1}{2}\epsilon_0 E^2$

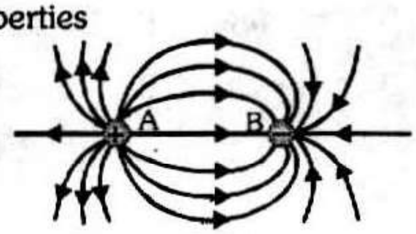
Electric lines of force

Electric lines of electrostatic field have following properties

(i) Imaginary

(ii) Can never cross each other

(iii) Can never be closed loops



$q_A > q_B$

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalised MKS system $(1/\varepsilon_o)$ electric lines are associated with unit charge, so if a body encloses a charge q , total lines of force associated with it (called flux) will be q/ε_o.

(v) Lines of force ends or starts normally at the surface of a conductor.

(vi) If there is no electric field there will be no lines of force.

(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity , crowded lines represent strong field while distant lines weak field.

(viii) Tangent to the line of force at a point in an electric field gives the direction of intensity.

KEY POINTS

- Electric field is always perpendicular to a conducting surface (or any equipotential surface).

  No tangential component of electric field on such surfaces.

- When a conductor is charged, the charge resides only on the surface.
- Charge density at convex sharp points on a conductor is greater. Lesser is radius of curvature at a convex part, greater is the charge density.
- For a conductor of any shape E (just outside)= $\frac{\sigma}{\varepsilon_o}$
- Potential difference between two points in an electric field does not depend on the path between them.
- Potential at a point due to positive charge is positive & due to negative charge is negative.
- Positive charge flows from higher to lower (i.e. in the direction of electric field) potential and negative charge from lower to higher (i.e. opposite to the electric field) potential.
- When $\vec{p} \parallel \vec{E}$ the dipole is in stable equilibrium
- When $\vec{p} \parallel (-\vec{E})$ the dipole is in unstable equilibrium

- When a charged isolated conducting sphere is connected to an uncharged small conducting sphere then potential become same on both sphere and redistribution of charge take place.
- Self potential energy of a charged conducting spherical shell = $\frac{KQ^2}{2R}$.
- Self potential energy of an insulating uniformly charged sphere= $\frac{3KQ^2}{5R}$
- A spherically symmetric charge {i.e ρ depends only on r} behaves as if its charge is concentrated at its centre (for outside points).
- **Dielectric strength of material :** The minimum electric field required to ionize the medium or the maximum electric field which the medium can bear without breaking down.
- The particles such as photon or neutrino which have no (rest) mass can never has a charge because charge cannot exist without mass.
- Electric charge is invariant because value of electric charge does not depend on frame of reference.
- A spherical body behaves like a point charge for outside points because a finite charged body may behave like a point charge if it produces an inverse square field.
- Any arbitrary displacement of charges inside a shell does not introduce any change in the electrostatic field of the outer space because a closed conducting shell divides the entire space into the inner and outer parts which are completely independent of one another in respect of electric fields.
- A charged particle is free to move in an electric field. It may or may not move along an electric line of force because initial conditions affect the motion of charged particle.
- Electrostatic experiments do not work well in humid days because water is a good conductor of electricity.
- A metallic shield in form of a hollow conducting shell may be built to block an electric field because in a hollow conducting shell, the electric field is zero at every point.

CAPACITOR & CAPACITANCE

A capacitor consists of two conductors carrying charges of equal magnitude and opposite sign. The capacitance C of any capacitor is the ratio of the charge Q on either conductor to the potential difference V between them $C = \frac{Q}{V}$

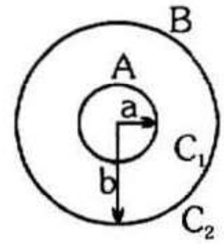
The capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference.

CAPACITANCE OF AN ISOLATED SPHERICAL CONDUCTOR

$C = 4\pi \epsilon_0 \epsilon_r R$ in a medium $C = 4\pi \epsilon_0 R$ in air

* This sphere is at infinite distance from all the conductors .

SPHERICAL CAPACITOR :



It consists of two concentric spherical shells as shown in figure. Here capacitance of region between the two shells is C_1 and that outside the shell is C_2. We have

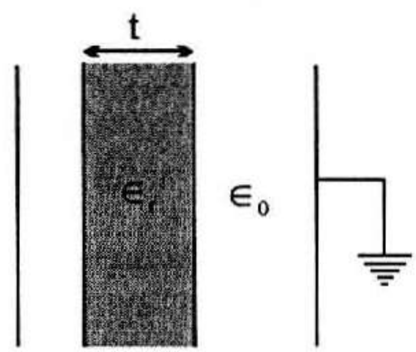
$C_1 = \frac{4\pi \epsilon_0 ab}{b-a}$ and $C_2 = 4\pi \epsilon_0 b$

PARALLEL PLATE CAPACITOR :

(i) UNIFORM DI-ELECTRIC MEDIUM :If two parallel plates each of area A & separated by a distance d are charged with equal & opposite charge Q, then the system is called a parallel plate capacitor & its capacitance is given by, $C = \frac{\epsilon_0 \epsilon_r A}{d}$ in a medium; $C = \frac{\epsilon_0 A}{d}$ with air as medium

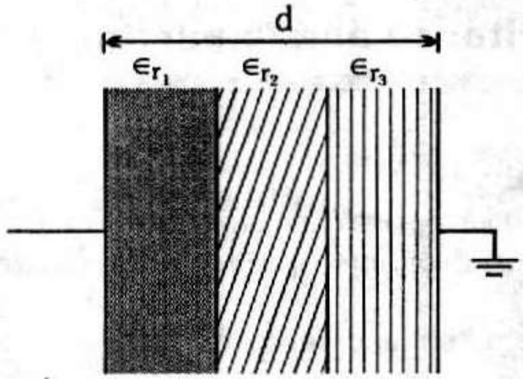
This result is only valid when the electric field between plates of capacitor is constant.

(ii) MEDIUM PARTLY AIR : $C = \frac{\epsilon_0 A}{d - \left(t - \frac{t}{\epsilon_r}\right)}$



When a di-electric slab of thickness t & relative permittivity ϵ_r is introduced between the plates of an air capacitor, then the distance between the plates is effectively reduced by $\left(t - \frac{t}{\epsilon_r}\right)$ irrespective of the position of the di-electric slab .

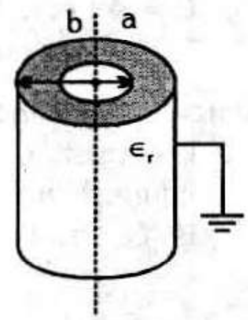
(iii) Composite Medium :



$$C = \frac{\epsilon_0 A}{\frac{t_1}{\epsilon_{r_1}} + \frac{t_2}{\epsilon_{r_2}} + \frac{t_3}{\epsilon_{r_3}}}$$

Cylindrical Capacitor :

It consists of two co-axial cylinders of radii a & b, the outer conductor is earthed. The di-electric constant of the medium filled in the space between the cylinders is ϵ_r.

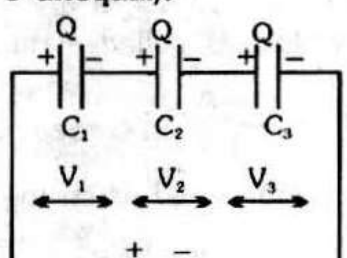


The capacitance per unit length is $C = \frac{2\pi\epsilon_0\epsilon_r}{\ell n\left(\frac{b}{a}\right)}$

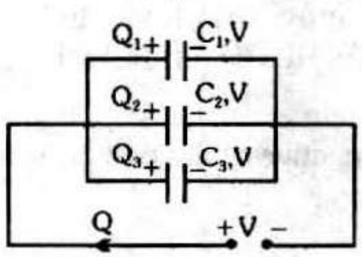
Combination Of Capacitors :

(i) Capacitors In Series :In this arrangement all the capacitors when uncharged get the same charge Q but the potential difference across each will differ (if the capacitance are unequal).

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}$$



(ii) Capacitors In Parallel : When one plate of each capacitor is connected to the positive terminal of the battery & the other plate of each capacitor is connected to the negative terminals of the battery, then the capacitors are said to be in parallel connection. The capacitors have the same potential difference, V but the charge on each one is different (if the capacitors are unequal). $C_{eq.} = C_1 + C_2 + C_3 + ... + C_n$.



Energy Stored In A Charged Capacitor :

Capacitance C, charge Q & potential difference V; then energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

This energy is stored in the electrostatic field set up in the di-electric medium between the conducting plates of the capacitor .

Heat produced in switching in capacitive circuit :

Due to charge flow always some amount of heat is produced when a switch is closed in a circuit which can be obtained by energy conservation as –

Heat = Work done by battery – Energy absorbed by capacitor.

- **Work done by battary to charge a capacitor** $W = CV^2 = QV = \frac{Q^2}{C}$

Sharing Of Charges :

When two charged conductors of capacitance C_1 & C_2 at potential V_1 & V_2 respectively are connected by a conducting wire, the charge flows from higher potential conductor to lower potential conductor, until the potential of the two condensers becomes equal. The common potential (V) after sharing of charges;

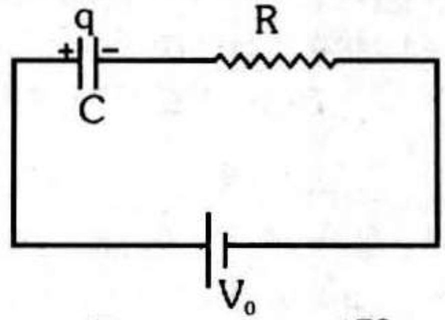
$$V = \frac{\text{net charge}}{\text{net capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}.$$

charges after sharing $q_1 = C_1V$ & $q_2 = C_2V$. In this process energy is lost in the connecting wire as heat.

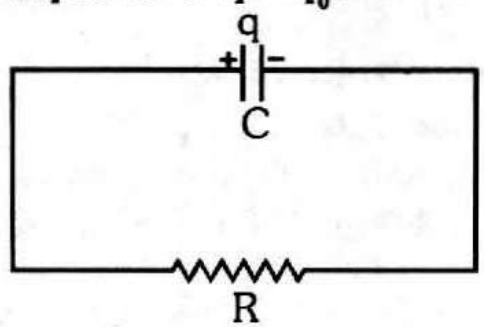
This loss of energy is $U_{\text{initial}} - U_{\text{final}} = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$.

- **Attractive force between capacitor plate** $F = \left(\frac{\sigma}{2\epsilon_0}\right)(\sigma A) = \frac{Q^2}{2\epsilon_0 A}$

- **Charging of a capacitor** : $q = q_0(1 - e^{-t/RC})$ where $q_0 = CV_0$



- **Discharging of a capacitor : $q = q_0e^{-t/RC}$**



KEY POINTS

- The energy of a charged conductor resides outside the conductor in its electric field, where as in a condenser it is stored within the condenser in its electric field.
- The energy of an uncharged condenser = 0.
- The capacitance of a capacitor depends only on its size & geometry & the dielectric between the conducting surface. (i.e. independent of the conductor, whether it is copper, silver, gold etc)
- The two adjacent conductors carrying same charge can be at different potential because the conductors may have different sizes and means difference capacitance.
- When a capacitor is charged by a battery, both the plates received charge equal in magnitude, no matter sizes of plates are identical or not because the charge distribution on the plates of a capacitor is in accordance with charge conservation principle.
- On filling the space between the plates of a parallel plate air capacitor with a dielectric, capacity of the capacitor is increased because the same amount of charge can be stored at a reduced potential.
- The potential of a grounded object is taken to be zero because capacitance of the earth is very large.