# ROTATIONAL MOTION

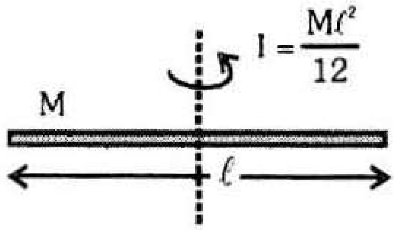
- **Angular velocity** $\vec{\omega} = \frac{d\vec{\theta}}{dt}$
- **Angular acceleration** $\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$
- **Angular momentum** $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$
- **Torque** $\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$
- **Rotational Kinetic energy** $K = \frac{1}{2} I\omega^2 = \frac{L^2}{2I}$
- **Rotational Power** $P = \vec{\tau}.\vec{\omega}$
- **For constant angular acceleration**

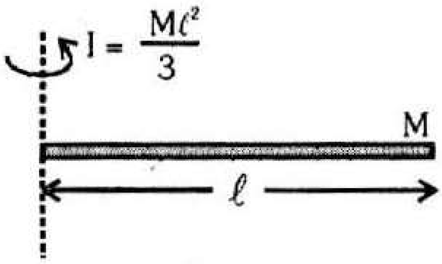
$$\omega = \omega_0 + \alpha t,\ \theta = \omega_0 t + \frac{1}{2}\alpha t^2,\ \ \omega^2 = \omega_0^2 + 2\alpha\theta,\ \ \theta_n = \omega_0 + \frac{\alpha}{2}(2n-1)$$

- **Moment of Inertia**
  - A tensor but for fixed axis it is a scalar
  - For discrete distribution of mass $I = m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots = \sum m_i r_i^2$
  - For continuous distribution of mass $I = \int dI = \int dm r^2$
- **Radius of gyration** $k = \sqrt{\frac{I}{M}}$
- **Theorems regarding moment of inertia**
  - Theorem of parallel axes $I_{axis} = I_{cm} + md^2$

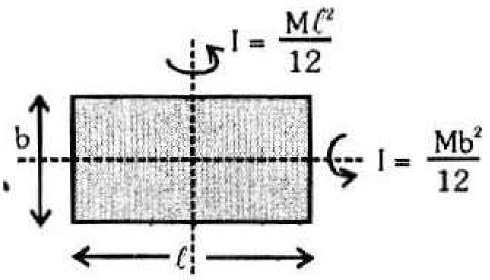
    where d is the perpendicular distance between parallel axes.
  - Theorem of perpendicular axes $I_z = I_x + I_y$

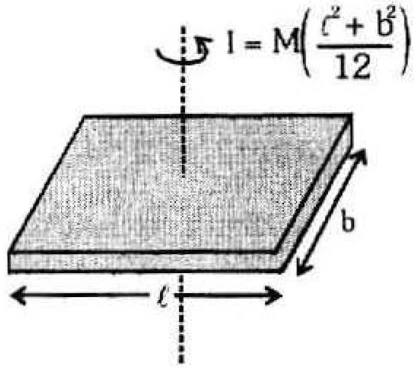
- **Rod**



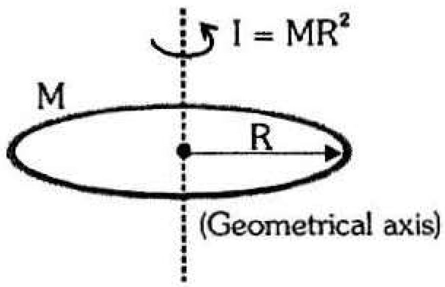




- **Rectangular Lamina**

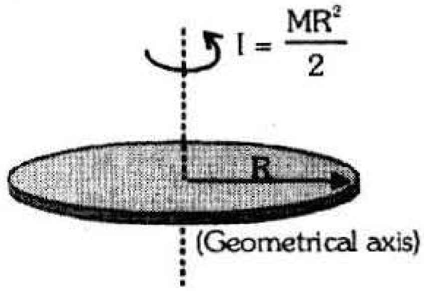


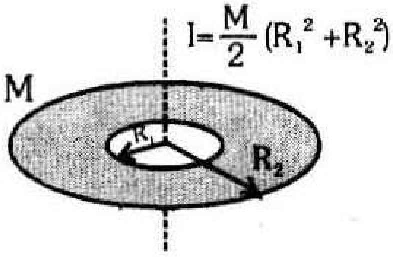

- **Ring :**



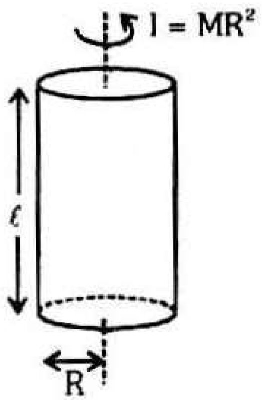

- **Disc :**

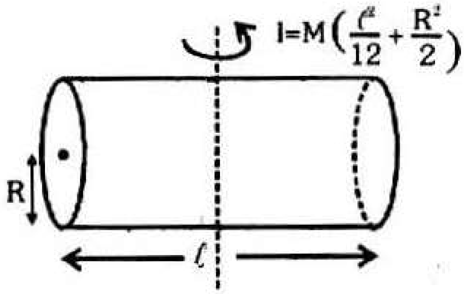



- **Circular Hollow Disk :**

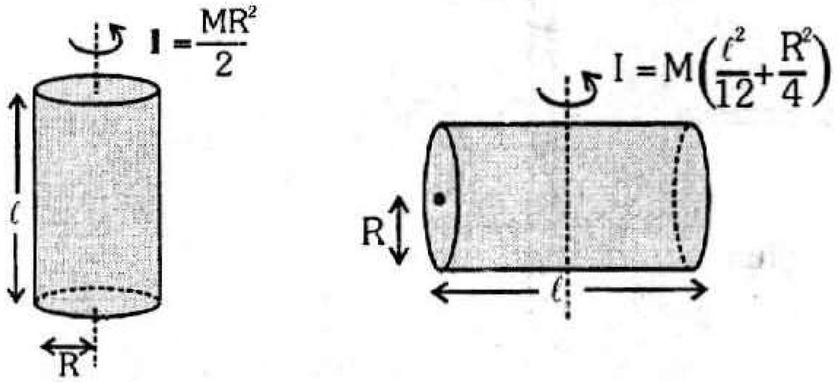



- **Hollow cylinder**

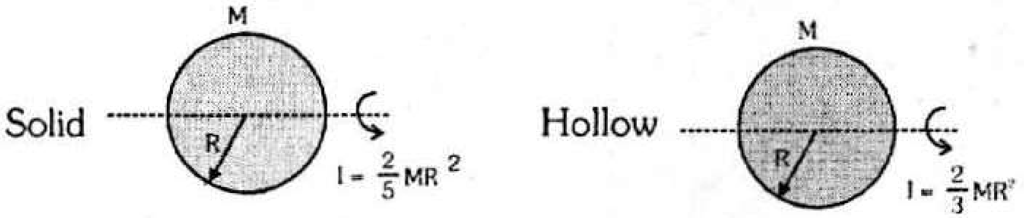


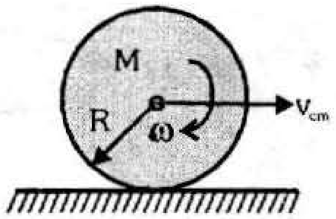
- **Solid cylinder**




- **Solid & Hollow sphere**




- **Rolling motion**




- Total kinetic energy = $\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{cm}\omega^2$

- Total angular momentum $= Mv_{CM}R + I_{cm}\omega$

- **Pure rolling (or rolling without slipping) on stationary surface**

- Condition : $v_{cm} = R\omega$

  In accelerated motion $a_{cm} = R\alpha$

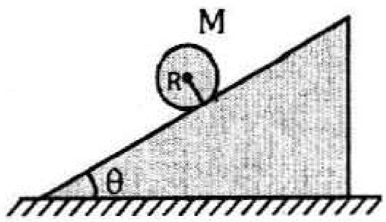
- If $v_{cm} > R\omega$ then rolling with forward slipping,

- If $v_{cm} < R\omega$ then rolling with backward slipping

- Total kinetic energy in pure rolling

$$K_{total} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}(Mk^2)\left(\frac{v_{cm}^2}{R^2}\right) = \frac{1}{2}Mv_{cm}^2\left(1 + \frac{k^2}{R^2}\right)$$

- **Pure rolling motion on an inclined plane**




  - Acceleration $a = \dfrac{g\sin\theta}{1+k^2/R^2}$
  - Minimum frictional coefficient $\mu_{min} = \dfrac{\tan\theta}{1+R^2/k^2}$

- **Torque** $\vec{\tau} = I\vec{\alpha} = I\dfrac{d\vec{\omega}}{dt} = \dfrac{d(I\vec{\omega})}{dt} = \dfrac{d\vec{L}}{dt} \text{ or } \dfrac{d\vec{J}}{dt}$

- **Change in angular momentum** $\Delta\vec{L} = \vec{\tau}\Delta t$

- **Work done by a torque** $W = \int \vec{\tau}\cdot d\vec{\theta}$

**KEY POINTS**

- A ladder is more apt to slip, when you are high up on it than when you just begin to climb because at the high up on a ladder the torque is large and on climbing up the torque is small.
- When a sphere is rolls on a horizontal table, it slows down and eventually stops because when the sphere rolls on the table, both the sphere and the surface deform near the contact. As a result the normal force does not pass through the centre and provide an angular deceleration.
- The spokes near the top of a rolling bicycle wheel are more blurred than those near the bottom of the wheel because the spokes near the top of wheel are moving faster than those near the bottom of the wheel.
- Instantaneous angular velocity is a vector quantity because infinitesimal angular displacement is a vector.
- The relative angular velocity between any two points of a rigid body is zero at any instant.
- All particles of a rigid body, which do not lie on an axis of rotation move on circular paths with centres at an axis of rotation.
- Instantaneous axis of rotation is stationary w.r.t. ground.
- Many greater rivers flow toward the equator. The sediment that they carry increases the time of rotation of the earth about its own axis because the angular momentum of the earth about its rotation axis is conserved.
- The hard boiled egg and raw egg can be distinguished on the basis of spinning of both.