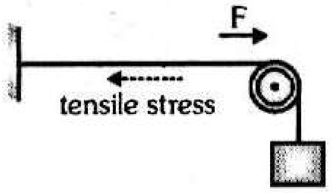
properties of matter

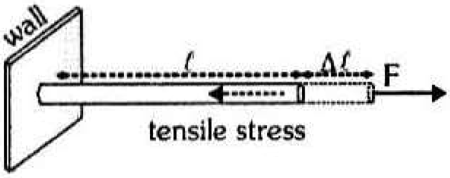
$$\textbf{STRESS} = \frac{\text{Internal restoring force}}{\text{Area of cross} - \text{section}} = \frac{F_{Res}}{A}.$$

There are three types of stress :-

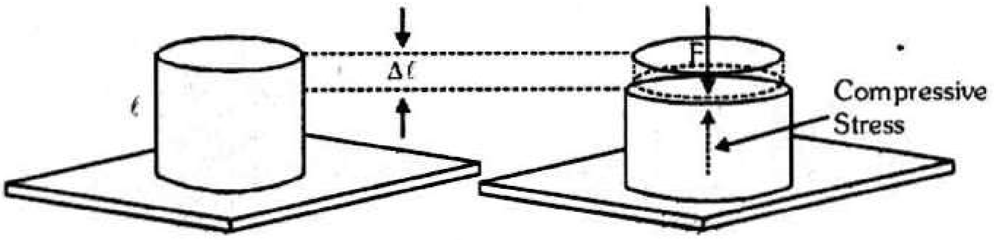
- **Longitudinal Stress**

(a) Tensile Stress:

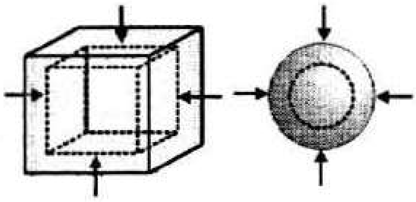




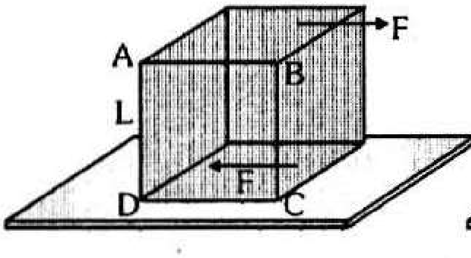
(b) Compressive Stress :

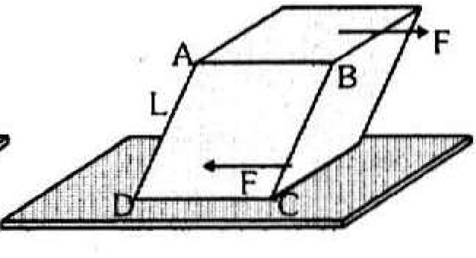


- **Volume Stress**



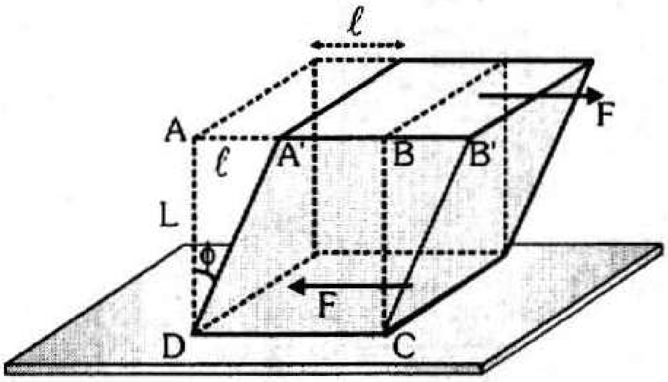
- **Tangential Stress or Shear Stress**





- $\textbf{Strain} = \dfrac{\text{Change in size of the body}}{\text{Original size of the body}}$
- $\textbf{Longitudinal strain} = \dfrac{\text{change in length of the body}}{\text{initial length of the body}} = \dfrac{\Delta L}{L}$
- $\textbf{Volume strain} = \dfrac{\text{change in volume of the body}}{\text{original volume of the body}} = \dfrac{\Delta V}{V}$

- **Shear strain** : $\tan \phi = \frac{\ell}{L}$ or $\phi = \frac{\ell}{L} = \frac{\text{displacement of upper face}}{\text{distance between two faces}}$



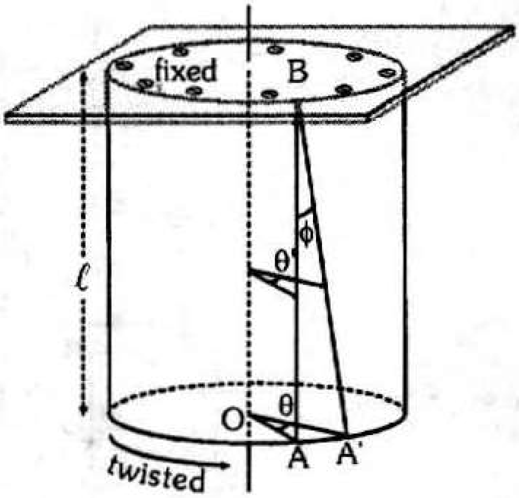
- **Relation between angle of twist (θ) & angle of shear (ϕ)**

AA' = r θ and Arc AA'=ℓϕ

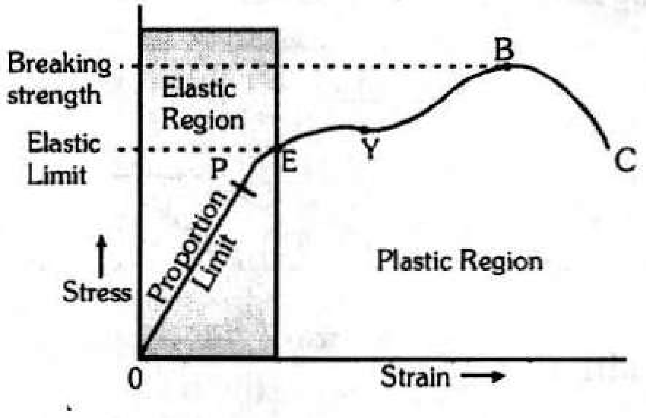
So $r\theta = \ell\phi \Rightarrow \phi = \frac{r\theta}{\ell}$

where θ = angle of twist,

ϕ = angle of shear



- **Stress – Strain Graph**



- **Hooke's Law** within elastic limit $\boxed{\text{Stress} \propto \text{strain}}$

- **Young's modulus of elasticity** $Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F\ell}{A\Delta\ell}$

- If L is the length of wire, r is radius and ℓ is the increase in length of the wire by suspending a weight Mg at its one end then Young's modulus of elasticity of the material of wire $Y = \frac{(Mg/\pi r^2)}{(\ell/L)} = \frac{MgL}{\pi r^2 \ell}$

- **Increment in length due to own weight** $\Delta\ell = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$

- **Bulk modulus of elasticity** $K = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{F/A}{\left(\frac{-\Delta V}{V}\right)} = \frac{P}{\left(\frac{-\Delta V}{V}\right)}$

- **Bulk modulus of an ideal gas is process dependence.**
  - For isothermal process PV = constant

    $\Rightarrow PdV + VdP = 0 \Rightarrow P = \frac{-dP}{dV/V}$ So bulk modulus = P

  - For adiabatic process PV^{γ} = constant $\Rightarrow \gamma PV^{\gamma-1}dV + V^{\gamma}dP = 0$

    $\Rightarrow \gamma PdV + VdP = 0 \Rightarrow \gamma P = \frac{-dP}{dV/V}$; So bulk modulus = γP

  - For any polytropic process PV^{n}= constant

    $\Rightarrow nPV^{n-1}dV + V^{n}dP = 0 \Rightarrow PdV + VdP = 0 \Rightarrow nP = \frac{-dP}{dV/V}$

    So bulk modulus = nP

- **Compressibility** $C = \frac{1}{\text{Bulk modulus}} = \frac{1}{K}$

- **Modulus of rigidity** $\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{(F_{\text{tangential}})/A}{\phi}$

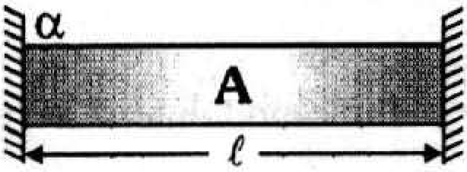
- **Poisson's ratio** $(\sigma) = \dfrac{\text{lateral strain}}{\text{Longitudinal strain}}$

- **Work done in stretching wire**

$$W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}:$$

$$W = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta\ell}{\ell} \times A \times \ell = \frac{1}{2} F \times \Delta\ell$$

- **Rod is rigidly fixed between walls**



  - Thermal Strain = $\alpha\Delta\theta$
  - Thermal stress = $Y\alpha\Delta\theta$
  - Thermal tension = $Y\alpha A\Delta\theta$

- **Effect of Temperature on elasticity**

When temperature is increased then due to weakness of inter molecular force the elastic properties in general decreases i.e. elastic constant decreases. Plasticity increases with temperature. For example, at ordinary room temperature, carbon is elastic but at high temperature, carbon becomes plastic. Lead is not much elastic at room temperature but when cooled in liquid nitrogen exhibit highly elastic behaviour.

For a special kind of steel, elastic constants do not vary appreciably temperature. This steel is called 'INVAR steel'.

- **Effect of Impurity on elasticity**

Y is slightly increase by impurity. The inter molecular attraction force inside wire effectively increase by impurity due to this external force can be easily opposed.

(B) HYDROSTATICS

- **Density** $= \dfrac{\text{mass}}{\text{volume}}$

- **Specific weight** $= \dfrac{\text{weight}}{\text{volume}} = \rho g$

- **Relative density** $= \dfrac{\text{density of given liquid}}{\text{density of pure water at } 4^\circ\text{C}}$

- **Density of a Mixture of substance in the proportion of mass**

  the density of the mixture is $\rho = \dfrac{M_1 + M_2 + M_3 \ldots}{\dfrac{M_1}{\rho_1} + \dfrac{M_2}{\rho_2} + \dfrac{M_3}{\rho_3} + \ldots}$

- **Density of a mixture of substance in the proportion of volume**

  the density of the mixture is $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}{V_1 + V_2 + V_3 + \ldots}$

- **Pressure** $= \dfrac{\text{normal force}}{\text{area}}$

- **Variation of pressure with depth**

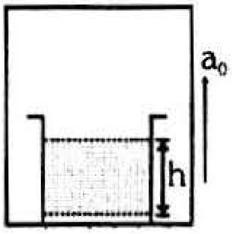
  Pressure is same at two points in the same horizontal level $P_1 = P_2$

  The difference of pressure between two points separated by a depth h

  $$(P_2 - P_1) = h\rho g$$

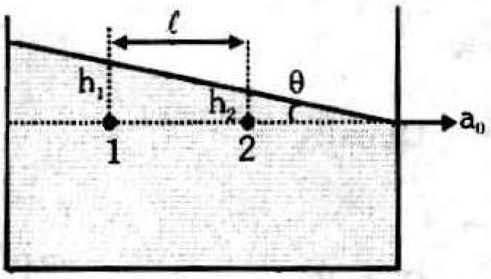
- **Pressure in case of accelerating fluid**

(i) **Liquid placed in elevator:** When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth h may be given by, $P = h\rho[g + a_0]$



(ii) Free surface of liquid in case of horizontal acceleration :

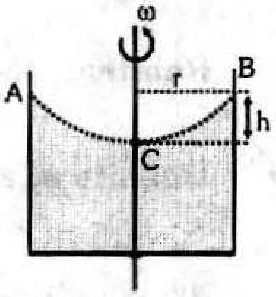
$$\tan\theta = \frac{ma_0}{mg} = \frac{a_0}{g}$$



If P_1 and P_2 are pressures at point 1 & 2 then

$$P_1 - P_2 = \rho g\,(h_1 - h_2) = \rho g \ell \tan\theta = \rho \ell a_0$$

(iii) Free surface of liquid in case of rotating cylinder



$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$

◆ **Pascal's Law**

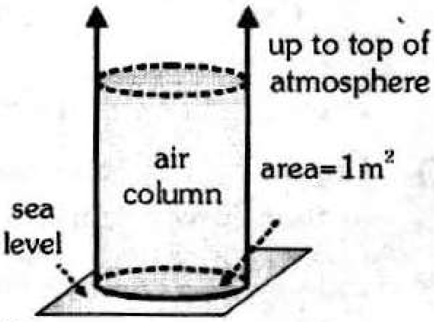
- The pressure in a fluid at rest is same at all the points if gravity is ignored.
- A liquid exerts equal pressures in all directions.
- If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude. *[for ideal fluids]*

◆ **Types of Pressure** : Pressure is of three types

(i) Atmospheric pressure (P_0) (ii) Gauge pressure (P_{gauge})

(iii) Absolute pressure ($P_{abs.}$)

◆ **Atmospheric pressure** : Force exerted by air column on unit cross–section area of sea level called atmospheric pressure (P_0)



$$P_0 = \frac{F}{A} = 101.3\ \text{kN/m}^2$$

$$\therefore\ P_0 = 1.013 \times 10^5\ \text{N/m}^2$$

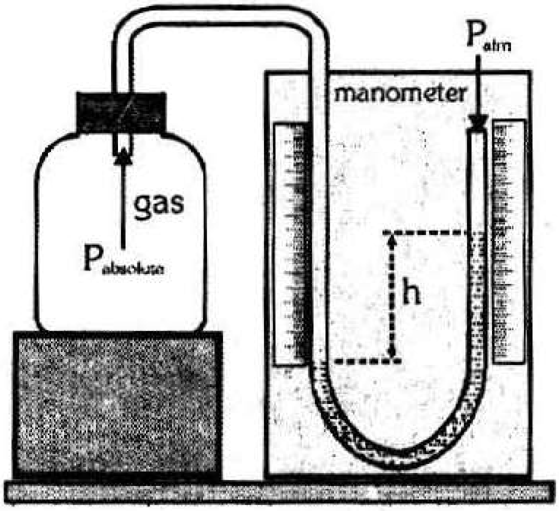
Barometer is used to measure atmospheric pressure.

Which was discovered by **Torricelli**.

Atmospheric pressure varies from place to place and at a particular place from time to time.

- **Gauge Pressure :**

  Excess Pressure ($P - P_{atm}$) measured with the help of pressure measuring instrument called Gauge pressure. $P_{gauge} = h\rho g$ or $P_{gauge} \propto h$



  Gauge pressure is always measured with help of **"manometer"**

- **Absolute Pressure :**

  Sum of atmospheric and Gauge pressure is called absolute pressure.

$$P_{abs} = P_{atm} + P_{gauge} \Rightarrow P_{abs} = P_o + h\rho g$$

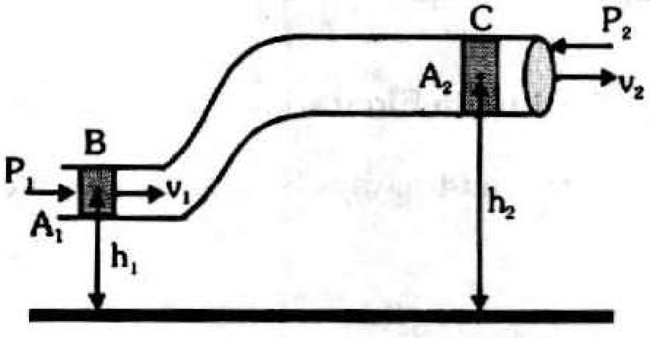
  The pressure which we measure in our automobile tyres is gauge pressure.

- **Buoyant force** = Weight of displaced fluid = $V\rho g$
- **Apparent weight** = Weight – Upthrust
- **Rotatory – Equilibrium in Floatation :** for rotational equilibrium of floating body the meta–centre must always be higher than the centre of gravity of the body.
- **Relative density of body** = $\dfrac{\text{Density of body}}{\text{Density of water}}$

(C) HYDRODYNAMICS

- **Steady and Unsteady Flow :** *Steady flow* is defined as that type of flow in which the fluid characteristics like velocity, pressure and density at a point do not change with time.
- **Streamline Flow :** In steady flow all the particles passing through a given point follow the same path and hence a unique line of flow. This line or path is called a *streamline*.
- **Laminar and Turbulent Flow :** *Laminar flow* is the flow in which the fluid particles move along well–defined streamlines which are straight and parallel.
- **Compressible and Incompressible Flow :** In *compressible flow* the density of fluid varies from point to point i.e. the density is not constant for the fluid whereas in *incompressible flow* the density of the fluid remains constant throughout.
- **Rotational and Irrotational Flow :** *Rotational flow* is the flow in which the fluid particles while flowing along path–lines also rotate about their own axis. In *irrotational flow* particles do not rotate about their axis.
- **Equation of continuity** $A_1v_1 = A_2v_2$ **Based** on conservation of mass
- **Bernoulli's theorem :** $\boxed{P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}}$

  Based on energy conservation



- **Kinetic Energy**

kinetic energy per unit volume $= \dfrac{\text{Kinetic Energy}}{\text{volume}} = \dfrac{1}{2}\dfrac{m}{V}v^2 = \dfrac{1}{2}\rho v^2$

- **Potential Energy**

Potential energy per unit volume $= \dfrac{\text{Potential Energy}}{\text{volume}} = \dfrac{m}{V}gh = \rho gh$

- **Pressure Energy**

Pressure energy per unit volume $= \dfrac{\text{Pressure energy}}{\text{volume}} = P$

- **For horizontal flow in venturimeter**

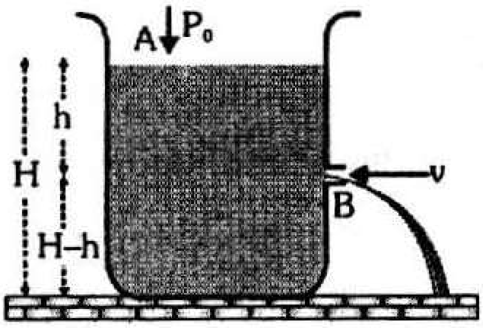
$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v^2 \Rightarrow v_1 = A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

- **Rate of flow :**

Volume of water flowing per second $Q = A_1v_1 = A_1A_2\sqrt{\dfrac{2gh}{A_1^2 - A_2^2}}$

- **Velocity of efflux** $v = \sqrt{2gh}$

- **Horizontal range** $R = 2\sqrt{h(H-h)}$



(D) SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum surface area. This property of liquid is called *surface tension.*

Intermolecular forces

(a) Cohesive force

The force acting between the molecules of one type of molecules of same substance is called cohesive force.

(b) Adhesive force

The force acting between different types of molecules or molecules of different substance is called adhesive force.

- ☐ Intermolecular forces are different from the gravitational forces and do not obey the inverse–square law
- ☐ The distance upto which these forces effective, is called.molecular range. This distance is nearly 10^{-9} m. Within this limit this increases very rapidly as the distance decreases.
- ☐ Molecular range depends on the nature of the substance

Properties of surface tension

- Surface tension is a scalar quantity.
- It acts tangential to liquid surface.
- Surface tension is always produced due to cohesive force.
- More is the cohesive force, more is the surface tension.
- When surface area of liquid is increased, molecules from the interior of the liquid rise to the surface. For this, work is done against the downward cohesive force.

Dependency of Surface Tension

- **On Cohesive Force :** Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.

- **On Impurities :** If the impurity is completely soluble then on mixing it in the liquid, its surface tension increases. e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively, e.g.

  (a) On mixing detergent in water its surface tension decreases.

  (b) Surface tension of water is more than (alcohol + water) mixture.

- **On Temperature**

  On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero.

  Note : Surface tension of water is maximum at 4°C

- **On Contamination**

  The dust particles or lubricating materials on the liquid surface decreases its surface tension.

- **On Electrification**

  The surface tension of a liquid decreases due to electrification because a force starts acting due to it in the outward direction normal to the free surface of liquid.

Definition of surface tension

The force acting per unit length of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is defined as surface tension.

☐ For floating needle $2T\ell \sin\theta = mg$

◆ **Required excess force for lift**

☐ Wire $F_{ex} = 2T\ell$	☐ Hollow disc $F_{ex} = 2\pi T(r_1 + r_2)$
☐ For ring $F_{ex} = 4\pi rT$	☐ Circular disc $F_{ex} = 2\pi rT$
☐ Square frame $F_{ex} = 8aT$	☐ Square plate $F_{ex} = 4aT$

- **Work** = surface energy = $T\Delta A$
  - ☐ Liquid drop $W = 4\pi r^2 T$
  - ☐ Soap bubble $W = 8\pi r^2 T$
- **Splitting of bigger drop into smaller droples** $R = n^{1/3} r$

Work done= Change in surface energy = $4\pi R^3 T\left(\frac{1}{r}-\frac{1}{R}\right) = 4\pi R^2 T\,(n^{1/3}-1)$

- **Excess pressure** $P_{ex} = P_{in} - P_{out}$
  - ☐ In liquid drop $P_{ex} = \frac{2T}{R}$
  - ☐ In soap bubble $P_{ex} = \frac{4T}{R}$

ANGLE OF CONTACT (θ_c)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the *angle of contact*.

The angle of contact depends the nature of the solid and liquid in contact.

- **Angle of contact** $\theta < 90° \Rightarrow$ concave shape, Liquid rise up

Angle of contact $\theta > 90° \Rightarrow$ convex shape, Liquid falls

Angle of contact $\theta = 90° \Rightarrow$ plane shape, Liquid neither rise nor falls

- **Effect of Temperature on angle of contact**

On increasing temperature surface tension decreases, thus $\cos\theta_c$ increases $\left[\because \cos\theta_c \propto \frac{1}{T}\right]$ and θ_c decrease. So on increasing temperature, θ_c decreases.

- **Effect of Impurities on angle of contact**

(a) Solute impurities increase surface tension, so $\cos\theta_c$ decreases and angle of contact θ_c increases.

(b) Partially solute impurities decrease surface tension, so angle of contact θ_c decreases.

- **Effect of Water Proofing Agent**

  Angle of contact increases due to water proofing agent. It gets converted acute to obtuse angle.

- **Capillary rise** $h = \dfrac{2T\cos\theta}{r\rho g}$

  - Zurin's law $h \propto \dfrac{1}{r}$

  - Jeager's method $T = \dfrac{rg}{2}(H\rho - hd)$

  - The height 'h' is measured from the bottom of the meniscus. However, there exist some liquid above this line also. If correction of this is applied

    then the formula will be $T = \dfrac{r\rho g\left[h + \dfrac{1}{3}r\right]}{2\cos\theta}$

- When two soap bubbles are in contact then radius of curvature of the common surface $r = \dfrac{r_1 r_2}{r_1 - r_2}(r_1 > r_2)$

- When two soap bubbles are combining to form a new bubble then radius of new bubble $r = \sqrt{r_1^2 + r_2^2}$

- Force required to separate two plates $F = \dfrac{2AT}{d}$

(E) VISCOSITY

- **Newton's law of viscosity** $F = \eta A \frac{\Delta v_x}{\Delta y}$

  - **SI UNITS :** $\frac{N \times s}{m^2}$ or deca poise
  - **CGS UNITS :** dyne–s/cm^2 or poise (1 decapoise = 10 poise)

- **Dependency of viscosity of fluids**

  On Temperature of Fluid

  (a) Since cohesive forces decrease with increase in temperature as increase in K.E.. Therefore with the rise in temperature, the viscosity of liquids decreases.

  (b) The viscosity of gases is the result of diffusion of gas molecules from one moving layer to other moving layer. Now with increase in temperature, the rate of diffusion increases. So, the viscosity also increases. Thus, the viscosity of gases increases with the rise of temperature.

  On Pressure of Fluid

  (a) The viscosity of liquids increases with the increase of pressure.

  (b) The viscosity of gases is practically independent of pressure.

  On Nature of Fluid

- **Poiseuillel's formula** $Q = \frac{dV}{dt} = \frac{\pi p r^4}{8\eta L}$

- **Viscous force** $F_v = 6\pi\eta r v$

- **Terminal velocity** $v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} \Rightarrow v_T \propto r^2$

- **Reynolds number** $R_e = \frac{\rho v d}{\eta}$

  $R_e < 1000$ laminar flow, $R_e > 2000$ turbulent flow