Basic Mathematics used in Physics

- **Quadratic equation**

Roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $x_1 + x_2 = -\frac{b}{a}$; Product of roots $x_1 x_2 = \frac{c}{a}$

- **Binomial theorem**

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + ...$

$(1-x)^n = 1 - nx + \frac{n(n-1)}{2}x^2 - \frac{n(n-1)(n-2)}{6}x^3 + ...$

If $x << 1$ then $(1+x)^n \approx 1 + nx$ & $(1-x)^n \approx 1 - nx$

- **Logarithm**

$\log mn = \log m + \log n$ $\qquad$ $\log \frac{m}{n} = \log m - \log n$

$\log m^n = n \log m$ $\qquad$ $\log_e m = 2.303 \log_{10} m$

$\log 2 = 0.3010$ $\qquad$ $\log 3 = 0.4771$

- **Componendo and dividendo theorem**

If $\frac{p}{q} = \frac{a}{b}$ then $\frac{p+q}{p-q} = \frac{a+b}{a-b}$

- **Arithmetic progression-AP**

$a, a+d, a+2d, a+3d, ...a+(n-1)d$ here d = common difference

Sum of n terms $S_n = \frac{n}{2}[2a+(n-1)d]$

Note : (i) $1+2+3+4+5....+ n = \frac{n(n+1)}{2}$

(ii) $1^2+2^2+3^2+...+ n^2 = \frac{n(n+1)(2n+1)}{6}$

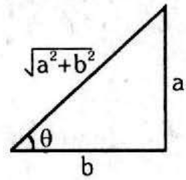
- **Geometrical progression-GP**

$a, ar, ar^2, ar^3, ...$ here, r = common ratio

Sum of n terms $S_n = \frac{a(1-r^n)}{1-r}$ $\qquad$ Sum of ∞ terms $S_\infty = \frac{a}{1-r}$

- **Trigonometry**

2π radian = 360° $\Rightarrow$ 1 rad = 57.3°



$\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{base}}{\text{hypotenuse}}$

$\tan\theta = \dfrac{\text{perpendicular}}{\text{base}}$ $\cot\theta = \dfrac{\text{base}}{\text{perpendicular}}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{base}}$ $\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{perpendicular}}$

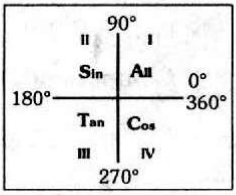
$\sin\theta = \dfrac{a}{\sqrt{a^2+b^2}}$ $\cos\theta = \dfrac{b}{\sqrt{a^2+b^2}}$ $\tan\theta = \dfrac{a}{b}$

$\text{cosec}\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \text{cosec}^2\theta$

$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$



$\sin 2A = 2\sin A\cos A$

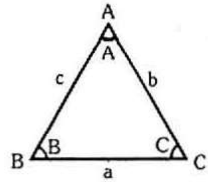
$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\sin(90°+\theta) = \cos\theta$	$\sin(180°-\theta) = \sin\theta$	$\sin(-\theta) = -\sin\theta$	$\tan(90°-\theta) = \cos\theta$
$\cos(90°+\theta) = -\sin\theta$	$\cos(180°-\theta) = -\cos\theta$	$\cos(-\theta) = \cos\theta$	$\cos(90°-\theta) = \sin\theta$
$\tan(90°+\theta) = -\cot\theta$	$\tan(180°-\theta) = -\tan\theta$	$\tan(-\theta) = -\tan\theta$	$\tan(90°-\theta) = \cot\theta$
$\sin(180°+\theta) = -\sin\theta$	$\sin(270°-\theta) = -\cos\theta$	$\sin(270°+\theta) = -\cos\theta$	$\sin(360°-\theta) = -\sin\theta$
$\cos(180°+\theta) = -\cos\theta$	$\cos(270°-\theta) = -\sin\theta$	$\cos(270°+\theta) = \sin\theta$	$\cos(360°-\theta) = \cos\theta$
$\tan(180°+\theta) = \tan\theta$	$\tan(270°-\theta) = \cot\theta$	$\tan(270°+\theta) = -\cot\theta$	$\tan(360°-\theta) = -\tan\theta$

- **sine law**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



- **cosine law**

$$\cos A = \frac{b^2+c^2-a^2}{2bc}, \cos B = \frac{c^2+a^2-b^2}{2ca}, \cos C = \frac{a^2+b^2-c^2}{2ab}$$

- **For small θ**

$\sin\theta \approx \theta$ $\quad \cos\theta \approx 1$ $\quad \tan\theta \approx \theta \sin\theta \approx \tan\theta$

- **Differentiation**

  - $y = x^n \rightarrow \frac{dy}{dx} = nx^{n-1}$
  - $y = \ell nx \rightarrow \frac{dy}{dx} = \frac{1}{x}$
  - $y = \sin x \rightarrow \frac{dy}{dx} = \cos x$
  - $y = \cos x \rightarrow \frac{dy}{dx} = -\sin x$
  - $y = e^{\alpha x+\beta} \rightarrow \frac{dy}{dx} = \alpha e^{\alpha x+\beta}$
  - $y = uv \rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
  - $y = f(g(x)) \Rightarrow \frac{dy}{dx} = \frac{df(g(x))}{dg(x)} \times \frac{d(g(x))}{dx}$
  - $y=k$ = constant $\Rightarrow \frac{dy}{dx} = 0$
  - $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

- **Integration**

  - $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
  - $\int \frac{1}{x} dx = \ell nx + C$
  - $\int \sin x dx = -\cos x + C$
  - $\int \cos x dx = \sin x + C$
  - $\int e^{\alpha x+\beta} dx = \frac{1}{\alpha} e^{\alpha x+\beta} + C$
  - $\int (\alpha x+\beta)^n dx = \frac{(\alpha x+\beta)^{n+1}}{\alpha(n+1)} + C$

- **Maxima & Minima of a function y = f(x)**
  - For maximum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = -ve$
  - For minimum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = +ve$

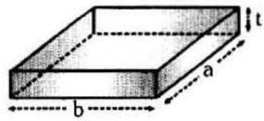
- **Average of a varying quantity**

If y = f(x) then $< y > = \bar{y} = \frac{\int_{x_1}^{x_2} y\,dx}{\int_{x_1}^{x_2} dx} = \frac{\int_{x_1}^{x_2} y\,dx}{x_2 - x_1}$

Formulae for determination of area

- Area of a square = $(\text{side})^2$
- Area of rectangle = length $\times$ breadth
- Area of a triangle = $\frac{1}{2} \times$ base $\times$ height
- Area of a trapezoid = $\frac{1}{2} \times$ (distance between parallel sides) $\times$ (sum of parallel sides)
- Area enclosed by a circle = πr^2 (r = radius)
- Surface area of a sphere = $4\pi r^2$ (r = radius)
- Area of a parallelogram = base $\times$ height
- Area of curved surface of cylinder = $2\pi r\ell$ (r = radius and ℓ = length)
- Area of whole surface of cylinder = $2\pi r(r + \ell)$ (ℓ = length)
- Area of ellipse = πab (a & b are semi major and semi minor axis respectively)
- Surface area of a cube = $6(\text{side})^2$
- Total surface area of a cone = $\pi r^2 + \pi r\ell$ where $\pi r\ell = \pi r\sqrt{r^2 + h^2}$ = lateral area

Formulae for determination of volume :



- Volume of a rectangular slab = length × breadth × height = abt
- Volume of a cube = $(\text{side})^3$
- Volume of a sphere = $\frac{4}{3}\pi r^3$ (r = radius)
- Volume of a cylinder = $\pi r^2 \ell$ (r = radius and ℓ = length)
- Volume of a cone = $\frac{1}{3}\pi r^2 h$ (r = radius and h = height)

KEY POINTS:

- To convert an angle from degree to radian, we have to multiply it by $\frac{\pi}{180^\circ}$ and to convert an angle from radian to degree, we have to multiply it by $\frac{180^\circ}{\pi}$.
- By help of differentiation, if y is given, we can find $\frac{dy}{dx}$ and by help of integration, if $\frac{dy}{dx}$ is given, we can find y.
- The maximum and minimum values of function $A\cos\theta + B\sin\theta$ are

  $\sqrt{A^2+B^2}$ and $-\sqrt{A^2+B^2}$ respectively.
- $(a+b)^2 = a^2 + b^2 + 2ab$ $\quad$ $(a-b)^2 = a^2 + b^2 - 2ab$

  $(a+b)(a-b) = a^2 - b^2$ $\quad$ $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

  $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

important notes